5. Multiplication table by $m(x) = x^3+x^2+1$ for $G(2^3)$

	0	1	x	$x+1$	x^2	x^2+1	x^2+x	x^2+x+1
0	0	0	0	0	0	0	0	0
1		1	x	$x+1$	x^2	x^2+1	x^2+x	x^2+x+1
x			x^2	x^2+x	x^2+1	x^2+x+1	1	$x+1$
$x+1$				x^2+1	1	x	x^2+x+1	x^2 ①
x^2					x^2+x+1	$x+1$ ②	x	x^2+x
x^2+1						x^2+x	x^2	1 ③
x^2+x							$x+1$	x^2+1 ④
x^2+x+1								x

Here are the calculations for labeled positions as example.

$x+1 \to 11$

$x^2+1 \to 101$

$x^2+x \to 110$

$x^2+x+1 \to 111$

$x^2 \to 100$

$x^3+x^2+1 \to 1101$

① $(x+1)\cdot(x^2+x+1)$

$$\begin{array}{r} 111 \\ 11 \\ \hline 111 \\ 111 \\ \hline 1001 \end{array}$$

$$\begin{array}{r|l} & 1 \\ \hline 1101 & 1001 \\ & 1101 \\ \hline & 100 \end{array} \leftarrow (x^2)$$

② $x^2(x^2+1)$

$$\begin{array}{r} 101 \\ 100 \\ \hline 000 \\ 000 \\ 101 \\ \hline 10100 \end{array}$$

$$\begin{array}{r|l} & 11 \\ \hline 1101 & 10100 \\ & 1101 \\ \hline & 1110 \\ & 1101 \\ \hline (x+1) \to & 11 \end{array}$$

③ $(x^2+1)(x^2+x+1)$

$$\begin{array}{r} 111 \\ 101 \\ \hline 111 \\ 000 \\ 111 \\ \hline 11011 \end{array}$$

$$\begin{array}{r|l} & 10 \\ \hline 1101 & 11011 \\ & 1101 \\ \hline (1) \to & 1 \end{array}$$

④ $(x^2+x)(x^2+x+1)$

$$\begin{array}{r} 111 \\ 110 \\ \hline 000 \\ 111 \\ 111 \\ \hline 10010 \end{array}$$

$$\begin{array}{r|l} & 11 \\ \hline 1101 & 10010 \\ & 1101 \\ \hline & 1000 \\ & 1101 \\ \hline (x^2+1) \to & 101 \end{array}$$

4.17

a) $x^3+1 \rightarrow 1001$

$x+1 \rightarrow 11$

```
      111
   ______
11 ) 1001
     11
     ---
      10
      11
      ---
       11
       11
       ---
        0
```

reducible

b) $x^3+x^2+1 \rightarrow 1101$

```
      101
   ______
11 ) 1101
     11
     ---
      01
      11
      ---
      10
```

~~no~~

irreducible

c) $x^4+1 \rightarrow 10001$

```
      1111
   _______
11 ) 10001
     11
     ---
     10
     11
     ---
      10
      11
      ---
       11
       11
       ---
        0
```

reducible

001

$GF(2) = \{0, 1, x, x+1\}$