COT 4500 Quiz #4 Solutions
Date: 3/2/2012

1) (25 pts) Use the Trapezoid Rule with n = 4 (4 trapezoids total), to approximate the following integral: $\int_{\pi/2}^{7\pi/6} e^{\sin x}\, dx$.

Solution
The evaluation points are x = π/2, x = 2π/3, x = 5π/6, x = π and x = 7π/6. The corresponding value of h is π/6, the height of each trapezoid. Here is the approximation:

$$\frac{\pi}{12}[e^{\sin\frac{\pi}{2}} + 2e^{sin2\pi/3} + 2e^{sin5\pi/6} + 2e^{sin\pi} + e^{sin7\pi/6}]$$
$$= \frac{\pi}{12}\left[e + 2e^{\frac{\sqrt{3}}{2}} + 2e^{\frac{1}{2}} + 2 + e^{-\frac{1}{2}}\right] \sim 3.502127162$$

2) (25 pts) Use Simpson's Rule with n = 4 (5 separate evaluation points, x_0 through x_4) to approximate the integral in question #2.

Solution
The evaluation points are x = π/2, x = 2π/3, x = 5π/6, x = π and x = 7π/6. The corresponding value of h is π/6. Here is the approximation:

$$\frac{\pi}{18}[e^{\sin\frac{\pi}{2}} + 4e^{sin2\pi/3} + 2e^{sin5\pi/6} + 4e^{sin\pi} + e^{sin7\pi/6}]$$
$$= \frac{\pi}{18}\left[e + 4e^{\frac{\sqrt{3}}{2}} + 2e^{\frac{1}{2}} + 4 + e^{-\frac{1}{2}}\right] \sim 3.513701341$$

3) (25 pts) Use the Gaussian Quadrature with n = 4 to approximate the integral $\int_{-1}^{3} e^{x^2} dx$. (Note: This approximation probably won't be great because this function can't be approximated well with a low order polynomial. Also, note that the bounds of the integral are not from -1 to 1.) The appropriate chart from the book is as follows:

Roots (for n=4)	Coeffcients (for n = 4)
.8611363116	.3478548451
.3399810436	.6521451549
-.3399810436	.6521451549
-.8611363116	.3478548451

Solution
Using a = -1, b = 3, we can map the integral as follows:

$$\int_{-1}^{3} e^{x^2} dx = \int_{-1}^{1} 2e^{(2t+1)^2} dt$$

Now, plug into the Gaussian Quadrature formula:

.3478548451*f(.8611363116) + .6521451549*f(.3399810436) +
.3478548451*f(-.8611363116) + .6521451549*f(-.3399810436)

where $f(t) = 2e^{(2t+1)^2}$, so

f(.861) ~ $2e^{2.72^2}$~ 1653.696286
f(.340) ~ $2e^{1.68^2}$~ ~ 16.81502134
f(-.340) ~ $2e^{0.320^2}$ ~ 1.107853396
f(-.861) ~ $2e^{(-.722)^2}$ ~ 1.684852027

Adding this all together, we get 587.5206653. (Note: The real answer is closer to 1446. I obtained this number by running both the Trapezoid and Simpson's Rule with n = 10000.) The issue here is that nearly all the area is under the curve from x = 2 to x = 3, but this portion of the curve is only weighted by .347, so to speak.

4) (25 pts) In class on Monday 27th, we coded up a version of the trapezoid rule for a double integral for an arbitrary number of intervals, *n*: $\int_a^b [\int_c^d f(x,y)dy]dx$. Write a C function that uses the Trapezoid Rule to approximate this given integral for a given function f that takes in two parameters, that you can call. Use the following prototype for your function:

<u>Solution</u>

```
double TrapRule(double a, double b, double c, double d, int n) {

    double sum = 0;
    double stepx = (b - a)/n;
    double stepy = (d - c)/n;

    int i,j;

    for (i=0; i<=n; i++) {
        for (j=0; j<=n; j++) {

            double coeff = 4;
            if (i == 0 || i == n) coeff /= 2;
            if (j == 0 || j == n) coeff /= 2;

            sum += (coeff*f(a+stepx*i, c+stepy*j));
        }
    }
    return sum;
}
```