1) (25 pts) Use the Trapezoid Rule with \( n = 4 \) (4 trapezoids total), to approximate the following integral: \( \int_{\pi/2}^{7\pi/6} e^{\sin x} \, dx \).

**Solution**
The evaluation points are \( x = \pi/2, x = 2\pi/3, x = 5\pi/6, x = \pi \) and \( x = 7\pi/6 \). The corresponding value of \( h \) is \( \pi/6 \), the height of each trapezoid. Here is the approximation:

\[
\frac{\pi}{12} \left[ e^{\sin \pi/2} + 2e^{\sin 2\pi/3} + 2e^{\sin 5\pi/6} + 2e^{\sin \pi} + e^{\sin 7\pi/6} \right] \\
= \frac{\pi}{12} \left[ e + 2e^{\sqrt{3}} + 2e^{1} + 2 + e^{-1/2} \right] \approx 3.502127162
\]

2) (25 pts) Use Simpson’s Rule with \( n = 4 \) (5 separate evaluation points, \( x_0 \) through \( x_4 \)) to approximate the integral in question #2.

**Solution**
The evaluation points are \( x = \pi/2, x = 2\pi/3, x = 5\pi/6, x = \pi \) and \( x = 7\pi/6 \). The corresponding value of \( h \) is \( \pi/6 \). Here is the approximation:

\[
\frac{\pi}{18} \left[ e^{\sin \pi/2} + 4e^{\sin 2\pi/3} + 2e^{\sin 5\pi/6} + 4e^{\sin \pi} + e^{\sin 7\pi/6} \right] \\
= \frac{\pi}{18} \left[ e + 4e^{\sqrt{3}} + 2e^{1} + 4 + e^{-1/2} \right] \approx 3.513701341
\]

3) (25 pts) Use the Gaussian Quadrature with \( n = 4 \) to approximate the integral \( \int_{-1}^{3} e^x \, dx \). (Note: This approximation probably won’t be great because this function can’t be approximated well with a low order polynomial. Also, note that the bounds of the integral are not from -1 to 1.) The appropriate chart from the book is as follows:

<table>
<thead>
<tr>
<th>Roots (for ( n=4 ))</th>
<th>Coefficients (for ( n = 4 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>.8611363116</td>
<td>.3478548451</td>
</tr>
<tr>
<td>.3399810436</td>
<td>.6521451549</td>
</tr>
<tr>
<td>-.3399810436</td>
<td>.6521451549</td>
</tr>
<tr>
<td>-.8611363116</td>
<td>.3478548451</td>
</tr>
</tbody>
</table>

**Solution**
Using \( a = -1, b = 3 \), we can map the integral as follows:

\[
\int_{-1}^{3} e^x \, dx = \int_{-1}^{1} 2e^{(2t+1)^2} \, dt
\]
Now, plug into the Gaussian Quadrature formula:

\[ .3478548451 \times f(.8611363116) + .6521451549 \times f(.3399810436) + .3478548451 \times f(-.8611363116) + .6521451549 \times f(-.3399810436) \]

where \( f(t) = 2e^{(2t+1)^2} \), so

\[
\begin{align*}
 f(.861) & \approx 2e^{2.72^2} \approx 1653.696286 \\
 f(.340) & \approx 2e^{1.68^2} \approx 16.81502134 \\
 f(-.340) & \approx 2e^{0.320^2} \approx 1.107853396 \\
 f(-.861) & \approx 2e^{(-.722)^2} \approx 1.684852027
\end{align*}
\]

Adding this all together, we get 587.5206653. (Note: The real answer is closer to 1446. I obtained this number by running both the Trapezoid and Simpson’s Rule with \( n = 10000 \).) The issue here is that nearly all the area is under the curve from \( x = 2 \) to \( x = 3 \), but this portion of the curve is only weighted by .347, so to speak.

4) (25 pts) In class on Monday 27\textsuperscript{th}, we coded up a version of the trapezoid rule for a double integral for an arbitrary number of intervals, \( n: \int_a^b \int_c^d f(x, y) \, dy \, dx \). Write a C function that uses the Trapezoid Rule to approximate this given integral for a given function \( f \) that takes in two parameters, that you can call. Use the following prototype for your function:

**Solution**

```c
double TrapRule(double a, double b, double c, double d, int n) {
    double sum = 0;
    double stepx = (b - a)/n;
    double stepy = (d - c)/n;

    int i, j;

    for (i=0; i<=n; i++) {
        for (j=0; j<=n; j++) {
            double coeff = 4;
            if (i == 0 || i == n) coeff /= 2;
            if (j == 0 || j == n) coeff /= 2;

            sum += (coeff*f(a+stepx*i, c+stepy*j));
        }
    }
    return sum;
}
```