Twelfth ACM Contest Finals (February 1988)

(Retyped with minor modifications)

Pluviometrics (Filename: VASE)

After the bottom dropped out of the hand-blown bud vase market, your employer purchased 100,000 of them for a pittance, intending to add a line of designer rain gauges to its geegaw catalog. After finding out that each vase has a shape uniquely its own, your employer handed you the problem of how to place the scale marks on them.

The marks on the rain gauge should be at the correct locations to indicate 0.1 inch, 0.2 inch, etc. Each 0.1 inch represents a volume of rainfall equal to 0.1 inch times the area of the opening at the top of the gauge.

The vases all have a single bulge at the base and flare toward the lip end. Since they were spun during blowing, their insides are surfaces of revolution about an axis perpendicular to a flat base. A tedious empirical study (in which, of course, least squares gave you fits) produced the result that the interior surfaces of the bud vases can be described closely enough by a function of four parameters that are easily determined for each vase. The parameters are:

h the height, in inches, measured from the inner surface of the bottom

d the diameter, in inches, of that round bottom

a, b $(0 < a < 1, 0 < b)$ parameters that describe the shape of the vase through the function $r(x)$, which gives the radius of the vase x inches above the inner bottom:

$$r(x) = \frac{d}{2}\sqrt{1 + a \sin\frac{2\pi x}{h} + b\frac{x}{h}}$$

Values of these four parameters will be determined for each vase and placed in a single record in the order h, d, a, b as fixed-point quantities separated by at least one blank. For each such record, the required program must print these parameters (suitably identified) and a table showing the height (in inches above the inside base) at which each tenth of an inch gradation in rainfall amount must be marked. The calculated height must be given to the nearest hundredth of an inch. Meniscus effects and refraction may be ignored. To avoid confusing the mark makers, no height exceeding h may appear.

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Possibly helpful facts:

$\pi \approx 3.14159265$

Volume of a cylinder of radius r and height h is $\pi r^2 h$

$\int \sin t \, dt = -\cos t + c$

Example:

h = 3.375 d = 0.715 a = 0.520 b = 0.133

Rain	Line
0.1	0.11
0.2	0.21
0.3	0.30
0.4	0.38
0.5	0.46
0.6	0.54
0.7	0.62
0.8	0.69
0.9	0.77
1.0	0.84
1.1	0.91

etc.