COT 4500, Spring 2007
S. Lang **Test #1** Name:______________________________

Instructions: The test consists of 7 questions, 4 pages, and 100 total points. Show your work in the space provided. You may use a calculator and the provided formula sheets for the test.

1. (10 pts.) Describe the pros and cons of the Method of False Position compared to Newton's Method if the initial approximations are chosen properly.

2. (10 pts.) Find the decimal value for the following 64-bit floating point number:
   1 10000010011 01101 (followed by 47 zeros)

3. (15 pts.) Show (prove) that the rate of convergence for the following computation is $O(x^3)$:
   $$\lim_{x \to 0} (\sin x - x) = 0$$

4. (15 pts.) Consider the function $f(x) = x - \cos x$ over the interval [0, 2].
   (a) Prove that the function $f(x)$ has a root in the interval [0, 2].

   (b) How many iterations are necessary to approximate a root in the interval [0, 2] with an absolute error $\leq 10^{-3}$ if the bisection method is used.

5. (15 pts.) Estimate the absolute error in approximating $\ell n(1.4)$ (ℓn is the natural logarithm function) based on the remainder term of the Taylor series if the third Taylor (Maclaurin) polynomial for the function $f(x) = \ell n(1+x)$ around the point $x = 0$ is used.

6. (15 pts.) Given the polynomial function $f(x) = x^3 + 2x^2 - 2$. Use Newton's method and initial approximation $p_0 = 2$ to compute the next approximate value p_1 if Horner's method is used in evaluating $f(x)$ and $f'(x)$.

7. (20 pts.) Consider the two functions $g_1(x) = \sqrt{\frac{4}{x}}$ and $g_2(x) = \frac{2x^3 + 4}{3x^2}$.

(a) Prove that the value $p = \sqrt[3]{4}$ is a fixed point for both functions $g_1(x)$ and $g_2(x)$.

(b) Suppose $p_0 = 1.5$ is used to approximate the fixed point $p = \sqrt[3]{4}$ using $g_1(x) = \sqrt{\frac{4}{x}}$.

Compute the next 4 approximations using Steffensen's method.

(c) When the fixed-point iteration algorithm is used to approximate the fixed point $p = \sqrt[3]{4}$ using an initial approximation $p_0 = 1.5$, explain why $g_2(x) = \frac{2x^3 + 4}{3x^2}$ converges faster than $g_1(x) = \sqrt{\frac{4}{x}}$. (**Hint**: Compute the derivative of the functions.)