COT 4500, Fall2006
S. Lang (12/08/2006) **Final Exam** Print Name _____
**Instructions:** There are 8 pages, 12 questions, and 100 total points. Write your answer neatly and concisely, and show all your work in the space provided.

1. (5 pts.) Find the value(s) of $\alpha$ that make the following matrix singular (**Hint**: Compute its determinant):

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & \alpha & 1 \\ 1 & 2\alpha & -2 \end{bmatrix}$$

2. (5 pts.) What is the main purpose of using partial pivoting when performing Gaussian elimination? What are the drawbacks of using partial pivoting?

**p.2**

3. (10 pts.) Consider the following matrix equation:

$AX = B$ where $A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & -3 \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, and $B = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$.

(a) Compute the inverse of matrix $A$.
(b) Use the result of Part (a) (i.e., $A^{-1}$) to solve for $X$ in the matrix equation.

**p.3**

4. (8 pts.) Find the norms $\|\cdot\|_1, \|\cdot\|_2$, and $\|\cdot\|_\infty$ for the matrix $A = \begin{bmatrix} 10 & -2 \\ 0 & 1 \end{bmatrix}$ (**Hint**: If $A$ is an $n \times n$ matrix and the entries of $A$ are given by $a_{ij}$ with $1 \le i \le n$ and $1 \le j \le n$, then $\|A\|_1 = \max\limits_{1 \le j \le n} \left\{ \sum\limits_{i=1}^{n} | a_{ij} | \right\}$ .)

5. (10 pts.) Compute the eigenvalues and associated eigenvectors for the following matrix:

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 4 \\ 0 & -1 & 2 \end{bmatrix}$$

**p. 4**

6. (12 pts.) Consider the following linear system of equations:

$$\begin{cases} 2x_1 - x_2 = 2 \\ x_1 + 2x_2 = -4 \end{cases}$$

(a) Find the first two iterations of the Jacobi method for solving this system assuming an initial approximation $x_1 = x_2 = 0$.
(b) Prove that the Jacobi method converges for solving this system, by computing the spectral radius of appropriate matrix.

(c) Suppose the infinity norm $\|\cdot\|_\infty$ is used in measuring the rate of convergence. Determine how many iterations are needed in applying the Jacobi method in order for the approximation error to be less than $10^{-3}$. (**Hint**: use Corollary 7.20 (b).)

**p. 5**

7. (10 pts.) Estimate the absolute error if the second Taylor polynomial for cos $x$ around $x = 0$ is used to approximate cos(0.02).

8. (5 pts.) Find the decimal value for the following 64-bit floating-point number

0 10000001001 10101 (followed by 47 zeros)

**p. 6**

9. (10 pts.) Consider the function $f(x) = \cos^2 x - x$.

(a) Find the first 3 iterations of applying Newton's root finding method for solving $f(x) = 0$ assuming $p_0 = 0$ is the initial approximation.

(b) In what sense do we say Newton's method converges quadratically?

(c) Name a general method that can be used to speed up the sequence of approximations produced by Newton's method.

**p. 7**

10. (5 pts.) Use the data from the following table and Newton's forward-difference formula to approximate $f(0.15)$:

| $x$ | $f(x)$ |
|---|---|
| 0.00 | 1.0 |
| 0.10 | 0.895 |
| 0.20 | 0.780 |

11. (8 pts.) Use appropriate formula and the following data to approximate $f''(0.10)$ (i.e., the second derivative of $f$ evaluated at 0.10). Estimate the absolute error of this approximation using the error formula if $f(x) = \cos x - x$.

| $x$ | $f(x)$ |
|---|---|
| 0.00 | 1.0 |
| 0.10 | 0.895 |
| 0.20 | 0.780 |

**p. 8**

12. (12 pts.) Approximate the integral $\int_1^3 \frac{e^x}{x} dx$ using each of the following methods:

(a) Simpson's rule:

(b) open Newton-Cotes formula with $n = 2$.

(c) Gaussian quadrature with $n = 2$.