CoT 4500, Spring 2009
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Answer key to Questions 3 and 4, Assignment #7

#3 (3pts) Maximal-column pivoting, 2-digit chopping arithmetic

$$\left[\begin{array}{ccc|c} -1 & 2 & 3 & -6 \\ 2 & -3 & -2 & 5 \\ 10 & 20 & 30 & -20 \end{array}\right] \text{ switch rows 1 and 3} \Rightarrow \left[\begin{array}{ccc|c} \textcircled{10} & 20 & 30 & -20 \\ 2 & -3 & -2 & 5 \\ -1 & 2 & 3 & -6 \end{array}\right] \begin{array}{l} \\ -0.2 \\ 0.1 \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 10 & 20 & 30 & -20 \\ 0 & \textcircled{-7} & -8 & 9 \\ 0 & 4 & 6 & -8 \end{array}\right] \begin{array}{l} \\ \\ 0.57 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 10 & 20 & 30 & -20 \\ 0 & -7 & -8 & 9 \\ 0 & 0 & 1.5 & -2.9 \end{array}\right]$$

Thus, $x_3 = \frac{-2.9}{1.5} = -1.9$

$x_2 = (9 + 8x_3)/-7 = 0.85$

$x_1 = (20 - 30x_3 - 20x_2)/10$

$= (-20 + 57 - 17)/10 = 2.0$

Answer: $\begin{bmatrix} 2.0 \\ 0.85 \\ -1.9 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

#4 (3pts) Scaled-column pivoting, 2-digit chopping arithmetic

$$\left[\begin{array}{ccc|c} -1 & 2 & 3 & -6 \\ 2 & -3 & -2 & 5 \\ 10 & 20 & 30 & -20 \end{array}\right]$$

Switch rows 1 and 2

Scale factors are:

$s_1 = 3$, $s_2 = |-3| = 3$, $s_3 = 30$

$\frac{|a_{11}|}{s_1} = \frac{1}{3} = 0.33$, $\frac{|a_{21}|}{s_2} = \frac{2}{3} = 0.66$, $\frac{|a_{31}|}{s_3} = \frac{10}{30} = 0.33$

$$\Rightarrow \left[\begin{array}{ccc|c} 2 & -3 & -2 & 5 \\ -1 & 2 & 3 & -6 \\ 10 & 20 & 30 & -20 \end{array}\right] \begin{array}{l} \\ 0.5 \\ \\ -5 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 2 & -3 & -2 & 5 \\ 0 & 0.5 & 2 & -3.5 \\ 0 & 35 & 40 & -45 \end{array}\right]$$

Switch rows 2 and 3

$\frac{|a_{22}|}{s_2} = \frac{0.5}{3} = 0.16$

$\frac{|a_{32}|}{s_3} = \frac{35}{30} = 1.1 > 0.16$

$$\Rightarrow \left[\begin{array}{ccc|c} 2 & -3 & -2 & 5 \\ 0 & 35 & 40 & -45 \\ 0 & 0.5 & 2 & -3.5 \end{array}\right] \begin{array}{l} \\ \\ -0.014 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 2 & -3 & -2 & 5 \\ 0 & 35 & 40 & -45 \\ 0 & 0 & 1.4 & -2.8 \end{array}\right]$$

Thus, $x_3 = -2.8/1.4 = -2.0$

$x_2 = (-45 - 40x_3)/35 = (-45 + 80)/35 = 1.0$

$x_1 = (5 + 2x_3 + 3x_2)/2 = (5 - 4 + 3)/2 = 2.0$

Answer: $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2.0 \\ 1.0 \\ -2.0 \end{bmatrix}$