COT 4500, Numerical Calculus — Assigned: April 17, 2009

S. Lang, Spring 2009 **Assignment #7** (30 pts.) — Due: Monday, April 27 in class

Instruction: The questions are taken from Exercise Sets 6.1 through 6.4 of the text. Write your answers clearly and show all relevant work and details.

Section 6.1:

1. (6 pts.) For each of the following liner systems obtain a solution by graphical methods if possible, and explain the solutions (or lack of) from a geometrical standpoint:

   (a) $\begin{cases} 2x - y = 3 \\ 3x + y = 7 \end{cases}$ (b) $\begin{cases} x + 3y = 3 \\ 2x - y = 2 \\ 3x + 2y = 4 \end{cases}$ (c) $\begin{cases} 2x - y = 3 \\ x + 3y = 1 \\ 3x + 2y = 4 \end{cases}$

2. (3 pts.) Use Gaussian elimination and backward substitution and two-digit chopping arithmetic **in every step of calculation** to solve the following linear system of equations written in augmented matrix format. Do not reorder equations (i.e., no pivoting). The exact solution is [2, −1, 1]. Show all steps of your work.

$$\left[\begin{array}{ccc|c} 2 & -1 & 1 & 6 \\ 1 & 3 & 1 & 0 \\ -1 & 5 & 4 & -3 \end{array}\right]$$

Section 6.2:

3. (3 pts.) Use Gaussian elimination and backward substitution, plus maximal-column pivoting and 2-digit chopping, to solve the following linear system of equations written in augmented matrix format. Show all steps of your work.

$$\left[\begin{array}{ccc|c} -1 & 2 & 3 & -6 \\ 2 & -3 & -2 & 5 \\ 10 & 20 & 30 & -20 \end{array}\right]$$

4. (3 pts.) Use Gaussian elimination and backward substitution, plus scaled-column pivoting and 2-digit chopping, to solve the same linear system of equations as in Question 3. Show all steps of your work.

Section 6.3:

5. (4 pts.) Determine if the following matrix is non-singular and find its inverse if exists (use exact values in all computations, i.e., use fractions instead of decimal values):

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & 2 \\ -1 & 2 & 2 \end{bmatrix}$$

6. (3 pts.) Use appropriate theorems and rules to prove that if $XA = AY = I$, where A, X, and Y are $n \times n$ matrices and I denotes the identity matrix of size $n \times n$, then $X = Y$, and therefore both X and Y are equal to A^{-1}, the inverse matrix for A. Explain each step of your proof.
   Hint: Consider the matrix product (XA)Y.

Section 6.4:

7. (4 pts.) Compute the determinant of the matrix in Question 5.
8. (4 pts.) Use Cramer's rule (p. 387 of the Text) and two-digit chopping arithmetic to solve the linear system in Question 2.