Assignment #7
(30 pts.) Due: Monday, April 27 in class

Instruction: The questions are taken from Exercise Sets 6.1 through 6.4 of the text. Write your answers clearly and show all relevant work and details.

Section 6.1:
1. (6 pts.) For each of the following linear systems obtain a solution by graphical methods if possible, and explain the solutions (or lack of) from a geometrical standpoint:

   \[
   \begin{align*}
   2x - y &= 3 \\
   3x + y &= 7
   \end{align*}
   \] (a)

   \[
   \begin{align*}
   x + 3y &= 3 \\
   2x - y &= 2 \\
   3x + 2y &= 4
   \end{align*}
   \] (b)

   \[
   \begin{align*}
   x + 3y &= 1 \\
   3x + 2y &= 4
   \end{align*}
   \] (c)

2. (3 pts.) Use Gaussian elimination and backward substitution and two-digit chopping arithmetic in every step of calculation to solve the following linear system of equations written in augmented matrix format. Do not reorder equations (i.e., no pivoting). The exact solution is \([2, -1, 1]\). Show all steps of your work.

   \[
   \begin{bmatrix}
   2 & -1 & 1 & 6 \\
   1 & 3 & 1 & 0 \\
   -1 & 5 & 4 & -3
   \end{bmatrix}
   \]

Section 6.2:
3. (3 pts.) Use Gaussian elimination and backward substitution, plus maximal-column pivoting and 2-digit chopping, to solve the following linear system of equations written in augmented matrix format. Show all steps of your work.

   \[
   \begin{bmatrix}
   -1 & 2 & 3 & -6 \\
   2 & -3 & -2 & 5 \\
   10 & 20 & 30 & -20
   \end{bmatrix}
   \]

4. (3 pts.) Use Gaussian elimination and backward substitution, plus scaled-column pivoting and 2-digit chopping, to solve the same linear system of equations as in Question 3. Show all steps of your work.

Section 6.3:
5. (4 pts.) Determine if the following matrix is non-singular and find its inverse if exists (use exact values in all computations, i.e., use fractions instead of decimal values):

   \[
   \begin{bmatrix}
   1 & 2 & -1 \\
   0 & 3 & 2 \\
   -1 & 2 & 2
   \end{bmatrix}
   \]

6. (3 pts.) Use appropriate theorems and rules to prove that if \(XA = AY = I\), where \(A\), \(X\), and \(Y\) are \(n \times n\) matrices and \(I\) denotes the identity matrix of size \(n \times n\), then \(X = Y\), and therefore both \(X\) and \(Y\) are equal to \(A^{-1}\), the inverse matrix for \(A\). Explain each step of your proof.

   Hint: Consider the matrix product \((X4)Y\).

Section 6.4:
7. (4 pts.) Compute the determinant of the matrix in Question 5.

8. (4 pts.) Use Cramer’s rule (p. 387 of the Text) and two-digit chopping arithmetic to solve the linear system in Question 2.