COT 4500, Numerical Calculus — Assigned: March 6, 2009

S. Lang, Spring 2009 — **Assignment #5** (30 pts.) — Due: Monday, March 23 in class

**Instruction**: Write your answers clearly and show all relevant work including details. You may use a calculator for the work.

**Section 3.1:**

1. (6 pts.) For each of the following functions let $x_0 = 1.0$, $x_1 = 1.5$ and $x_2 = 1.8$. Construct Lagrange interpolating polynomials of degree (at most) two using $x_0$, $x_1$, and $x_2$, to approximate $f(1.4)$, and find the absolute error in each case using 6-digit chopping for the final answer:
   (a) $f(x) = \sin x + \cos x$
   (b) $f(x) = x \ln x$
2. (4 pts.) Use the error bound formula of Theorem 3.3 to estimate the errors for the approximations used in both (a) and (b) of Question #1.

**Section 3.2:**

3. (4 pts.) Use the divided-difference formula of Equation 3.10 to construct interpolating polynomials of degree three for the following data, and find the approximate values using the interpolating polynomials in each case: Approximate $f(0.25)$ if $f(0.0) = 0.00000$, $f(0.20) = 0.202710$, $f(0.40) = 0.422793$, and $f(0.60) = 0.684136$ assuming 6-digit chopping arithmetic is used.
4. (6 pts.) Repeat Question #3 by using the Newton forward-difference formula (Equation 3.12, or equivalently, Equation 3.11) and Newton backward-difference formula (Equation 3.13) to construct the interpolating polynomial.

**Section 3.3:**

5. (5 pts.) Use Theorem 3.9 to construct the Hermite polynomial of degree three using the following data with 6-digit chopping:

| $x$ | $f(x)$ | $f'(x)$ |
|---|---|---|
| 1.1 | 1.96475 | 4.86028 |
| 1.2 | 2.57215 | 7.61596 |

6. (5 pts.) Use the following Hermite interpolating polynomial formula of the textbook (p. 133)

$$H_{2n+1}(x) = f[z_0] + \sum_{k=1}^{2n+1} f[z_0, \ldots, z_k](x - z_0)(x - z_1)\cdots(x - z_{k-1}).$$

   and the data from Question #5 to approximate $f(1.16)$, and compute the absolute error assuming the function $f(x) = \tan x$ and using 6-digit chopping arithmetic.