COT 4500, Numerical Calculus — Assigned: February 16, 2009

S. Lang, Spring 2009 **Assignment #4** (30 pts.) — Due: Monday, February 23 in class

Instruction: Write your answers clearly and show all relevant work including details. You may use a calculator and **5-digit chopping** unless specified otherwise.

Note the typo in Question 3(c) corrected on 2/16/2009.

Section 2.3 :

1. (3 pts.) Let $f(x) = e^{2x} - x^2 + \cos x$. Use Newton's method to approximate a root to the equation $f(x) = 0$ after 3 iterations, by using the initial approximation $p_0 = -1.0$.
2. (4 pts.) Define the same function $f(x) = e^{2x} - x^2 + \cos x$ as in Question 1. Find an approximation to a root for equation $f(x) = 0$ with $p_0 = -1.0$ and $p_1 = 0.0$ after 3 iterations, based on (a) the Secant method; and (b) the method of false position.

Section 2.4:

3. (7 pts.) Define $f(x) = x^2 \ln(1+x)$ for $x > -1$.
   (a) Show that $x = 0$ is a zero of multiplicity 3 for the function $f(x)$. **(Hint: use Theorem 2.11 and prove that $f(0) = f'(0) = f''(0) = 0$.)**
   (b) Use Algorithm 2.3 (Newton's method) posted at the textbook's website http://www.as.ysu.edu/~faires/Numerical-Analysis/DiskMaterial/programs/Java/JavaPrograms.htm to compute a root for the equation $f(x) = x^2 \ln(1+x) = 0$ using an initial approximation $p_0 = 0.5$ and an accuracy to within 10^{-5}.
   (c) Redo Part (**b**) by computing an approximation to the root for the equation $f(x) = x^2 \ln(1+x) = 0$ using an initial approximation $p_0 = 0.5$ based on the modified Newton-Raphson method described in Equation 2.11 of the text. Compare this algorithm's speed of convergence to the speed of Newton's method of Part (a).
4. (4 pts.) Suppose the sequence p_n converges to p as $n \to \infty$ and $\dfrac{|p_{n+1} - p|}{|p_n - p|^3} \le 0.5$ for all $n \ge 1$. (a) Show that $|p_4 - p| \le |p_1 - p|^{27}\,(0.5)^{1+3+3^2}$; and (b) in general, find a relationship between $|p_n - p|$ vs. $|p_1 - p|$ for any $n \ge 2$.

Section 2.5:

5. (4 pts.) Let $f(x) = x^2 \ln(1+x) = 0$ as in Question 3. Apply Steffensen's technique to Newton's method and compute approximation $p_0^{(2)}$ as done in Example 2 of this section (p. 85) of the text.

Section 2.6:

6. (8 pts.) Let $f(x) = x^3 - 9x^2 - 9x - 10$.
   (a) Apply Newton's method and Horner's method using synthetic division to approximate the real root of $f(x) = 0$ after 3 iterations, using initial approximation $p_0 = 8.0$. Show your work in each of the steps.
   (b) Use Algorithm 2.8 (Muller's method) posted at the textbook's website http://www.as.ysu.edu/~faires/Numerical-Analysis/DiskMaterial/programs/Java/Algo28.htm to approximate the two complex roots for $f(x) = 0$ within 10^{-5}, using the initial approximations.0, 1, and 2. In addition, find the exact values of the two complex roots.(using the quadratic formula).