COT 4500, Numerical Calculus | Assigned: Feb. 6, 2009

S. Lang, Spring 2009 **Assignment #3** (30 pts.) | Due: Monday, Feb. 16 in class

Note: Questions 3 and 4 added 2/0/2009.

Instruction: Write your answers clearly and show all relevant work including details. You may use a calculator for the work.

Exercise Set for Section 1-3 (pp. 36 – 37):

1. (8 pts.) Find the rate of convergence for each of the following question:
   (a) $\lim_{n\to\infty} n\ln(1+\frac{1}{2n})$, using the big-O notation in terms of the parameter *n*.
   (b) $\lim_{h\to 0}\frac{1+\sin h - e^h}{2h}$, using the big-O notation in terms of the parameter *h*.

Exercise Set for Section 2-1 (pp. 51 – 52):

2. (6 pts.) Apply the Bisection method to find a solution accurate to within 10^{-5} for the equation $x^2\cos x + x - 1 = 0$, $1 \le x \le 3$, by using the Java program of Algorithm 2.1 posted at the textbook's website (http://www.as.ysu.edu/~faires/Numerical-Analysis/DiskMaterial/programs/Java/JavaPrograms.htm) for the calculations. In addition, use Theorem 2.1 of the textbook to estimate how many iterations are needed to achieve this level of accuracy.

Exercise Set for Section 2-2 (pp. 61 – 63):

3. (10 pts.) Define $g(x) = 1.5 + \sin x - x$. Answer all parts given below:
   (a) Use Theorem 2.2 to show that $g(x) = 1.5 + \sin x - x$ has a unique fixed point on [0.5, 1.5].
   (b) Sketch the graph for $y = g(x)$ over the interval [0.5, 1.5].
   (c) Use fixed-point iteration and the initial approximation $p_0 = 1.0$ to find an approximation to the fixed point that is accurate to within 10^{-7} by using the Java program Algorithm 2.2 posted at the textbook's website http://www.as.ysu.edu/~faires/Numerical-Analysis/DiskMaterial/programs/Java/JavaPrograms.htm.
   (d) Use Corollary 2.4 to estimate the number of iterations required to achieve the accuracy of 10^{-7}, and compare this theoretical estimate to the number actually needed from Part (c).
4. (6 pts.) Define $g(x) = \frac{x+2}{x+1}$. Answer all parts given below:
   (a) Use Theorem 2.3 to determine an interval [*a*, *b*] so that $g(x)$ has a fixed point and that the fixed-point iteration algorithm 2.2 converges. (Hint: Choose an interval around a fixed point found in Part (b).)
   (b) Determine the exact value of the fixed point (or points) by solving the equation $g(x) = x$.