Exercise Set for Section 1-2 (pp. 26 – 29):

1. (4 pts.) Find the value for \( p = \sqrt{242} \) using 5-digit rounding arithmetic. Find the (largest) interval in which an approximation \( p^* \) to \( p \) lies if (a) the absolute error is within \( 10^{-5} \); and (b) the relative error is within \( 10^{-5} \). Use 5-digit rounding arithmetic for this question.

2. (3 pts.) Perform the following calculation (a) exactly; (b) using 3-digit chopping arithmetic; (c) using 3-digit rounding arithmetic: \[ \frac{17}{11} \cdot \frac{11}{2} \] For Parts (b) and (c), calculate each fraction separately then multiply the results, using chopping or rounding arithmetic, respectively, in each step.

3. (3 pts.) The value \( e^{-1} \) can be computed by the formula: \[ e^{-1} = \sum_{n=2}^{\infty} \frac{(-1)^n}{n!} \] (This can be verified by using the Taylor series for \( f(x) = e^x \) about the point \( x_0 = 0 \) then substituting \( x = -1 \).) Compute the absolute and relative error if \( e^{-1} \) is approximated by \[ e^{-1} = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} \] where all calculations are done using 5-digit chopping arithmetic.

4. (8 pts.) Let \( f(x) = \frac{e^{-2x} - 1}{x} \) for \( x \neq 0 \); and \( f(0) = -2 \). (It can be proven that \( f(x) \) is infinitely differentiable for all \( x \).) Answer each of the following questions:
   (a) Find \( \lim_{x \to 0} f(x) \). (Hint: Use L’Hopital’s rule.)
   (b) Use 3-digit chopping arithmetic to calculate \( f(0.05) \). That is, use a calculator to calculate \( e^{-2(0.05)} - 1 \) using 3-digit chopping, then divide the result by 0.05 and report the answer using 3-digit chopping.
   (c) Replace \( f(x) \) by its third-degree Maclaurin polynomial and redo Part (b).
   (d) The exact value for \( f(0.05) \) is \(-1.90325\) using 6-digit chopping. Find the absolute errors for the values computed in Parts (b) and (c).

5. (6 pts.) Given the following 64-bit floating-point value in IEEE 754-1985 format: 1 00000110011 010101 (followed by 46 zeros).
   (a) Find the decimal value (and show the steps of your work).
   (b) Conversely, convert the answer of Part (a) back to the 64-bit floating-point format (and show the steps of your work).

6. (6 pts.) Suppose \( f \in C^4[a-h, a+h] \) where \( h > 0 \) is a small number. An approximation formula for \( f'(a) \) is \( f'(a) \approx \frac{f(a+h) - f(a-h)}{2h} \) when \( h \) is sufficiently small.
   (a) Use Taylor’s Theorem to find the error term in the above approximation. **Hint:** Apply Taylor’s Theorem to find the second-degree Taylor polynomial plus the error term, \( P_2(x) + R_2(x) \) for \( f(x + h) \), about the point \( x_0 = a \). Similarly, find the second-degree Taylor polynomial plus the error term, \( Q_2(x) + S_2(x) \) for \( f(x-h) \), about the point \( x_0 = a \). Calculate \[ \frac{f(a+h) - f(a-h)}{2h} \] then show that \( f'(a) = \frac{f(a+h) - f(a-h)}{2h} - \frac{h^2}{6} f^{(3)}(\xi) \) where \( a-h < \xi < a+h \). Note that the Intermediate Value Theorem of the text is useful in the last step of formula derivation.
(b) If \( h = 10^{-5} \) and \( f(x) = \sin x \), where \( a = 1.0 \). Find the exact value for \( f'(a) \) and the absolute error using the approximate value given above assuming 6-digit chopping arithmetic in the calculations.

**Note on Question 1(a) of Assignment #1:**

Consider the function \( f(x) = \frac{\sin x}{x + 1} - 0.4 \) defined on \( x \in (-1, \infty) \). Find the value for \( \lim_{x \to \infty} f(x) \).

**Answer:** Since \(|\sin x| \leq 1\) and \( \frac{1}{x + 1} \to 0\) as \( x \to \infty \), \( \lim_{x \to \infty} f(x) = \lim_{x \to \infty} \left( \frac{\sin x}{x + 1} - 0.4 \right) = 0 - 0.4 = -0.4 \).