COT 4500, Numerical Calculus — Assigned: January 12, 2009

S. Lang, Spring 2009 — **Assignment #1** (30 pts.) — Due: Wednesday, January 21 in class

Instruction: Write your answers clearly and show all relevant work including details. You may use a calculator for the work **but do not depend your work solely on graphs produced by a graphing calculator.**

Note (added after class on 1/12): Change in Question 1 and details on Question 2.

Further Notes (1/16/2009): (1) Use 6-digit chopping arithmetic in your calculations; that is, keep 6 decimal places (and drop anything beyond) when a number is in normalized form ($0.d_1d_2d_3d_4d_5d_6 \times 10^e$). For example, π = 3.14159265358979323846264338327950… but use π = 3.14159 in 6-digit chopping arithmetic. (2) For this assignment you may use the Java program posted at the author's website (the bisection method at http://www.as.ysu.edu/~faires/Numerical-Analysis/DiskMaterial/programs/Java/Algo21.htm) to find roots for equation $f(x) = 0$ if the function $f(x)$ is continuous over interval $[a, b]$ and $f(a)\,f(b) \le 0$. In that case the Intermediate Value Theorem implies there is a root c, $a \le c \le b$, such that $f(c) = 0$.

Exercise Set for Section 1-1 (pp. 14 – 15):

1. (6 pts.) Consider the function $f(x) = \dfrac{\sin x}{x+1} - \mathbf{0.4}$ defined on $x \in (-1, \infty)$.
   (a) Find the value for $\lim_{x \to -1^+} f(x)$ where $x \to -1^+$ means approaching -1 from the rightside, and find the value for $\lim_{x \to \infty} f(x)$.
   (b) Determine an (i.e., at least one) interval $[n, n + 1]$ where n is an integer and $f(x) = 0$ has at least one solution in the interval. Explain which theorem or theorems are used in deriving your result and the necessary conditions in order for the theorem(s) to apply.
2. (6 pts.) Find $\max_{a \le x \le b} |f(x)|$ for the following functions and the intervals (a calculator is useful in finding specific values for sine and cosine functions but not for graphing):
   (a) $f(x) = \dfrac{e^x}{x} - 10$, $x \in [1, 5]$.
   (b) $f(x) = 2x \cos x + 1$, $x \in [0, 2]$.
3. (8 pts.) Find the third Taylor polynomial $P_3(x)$ for function $f(x) = \sqrt{x^2 - 1}$ about $x_0 = 2$. Approximate $\sqrt{3.41}$, $\sqrt{3.0401}$, and $\sqrt{2.9601}$ using $P_3(x)$, and find the actual errors using a calculator. (Hint: when $x = 2.1$, $x^2 - 1 = 3.41$; when $x = 2.01$, $x^2 - 1 = 3.0401$; etc.)
4. (6 pts.) Let $f(x) = \ln(x^2 + 1)$ and $x_0 = 0$.
   (a) Find the third Taylor polynomial $P_3(x)$, and use it to approximate $f(0.4)$.
   (b) Use the error formula in Taylor's Theorem to find an upper bound for the error $|f(0.4) - P_3(0.4)|$. Compute the actual error using a calculator.
5. (4 pts.) Let $f(x) = \sin(1 + x)$ and $x_0 = 0$. Find the nth Taylor polynomial $P_n(x)$ for $f(x)$ about x_0. Find a value of n necessary for $P_n(x)$ to approximate $f(x)$ to within 10^{-6} on [0, 0.5].