COT 4210 Quiz #4 Part B: Class NP 4/22/2021 Solutions

1) (10 pts) Let 4COLOR = { <G> | G is colorable with 4 colors. }. It can be proven that FOUR-COLOR is an NP-Complete language.

Specifically, we can show that 3COLOR $\leq_P$ 4COLOR. One the reduction is shown, the results follows.

For this question, show the polynomial time reduction between the two problems. (Hint: This reduction isn't as difficult as it may initially seem.)

Solution

To do the reduction, we must take as input a graph G and produce a new graph G', such that, if and only if G is 3 colorable, G' is 4 colorable.

Given a graph G = (V, E), the set of vertices V and set of edges E which comprise G, create the set V' = $V \cup \{x\}$, where x is a vertex not in V, and create a set E' = $E \cup \{(u, x) | u \in V\}$. The desired graph G' = {V', E'}.

Intuitively, to create G' from G, add a new vertex and connect it to all vertices from G. If G can be colored in 3 colors, then we can simply add a 4th color to color the newly added vertex. Similarly, if G' can be colored in 4 colors, we know that no other vertex can share the color assigned to x, because x is connected to all of the other vertices. Thus, all the vertices in V must be assigned 3 colors in the 4-coloring of G'. But this coloring would infer a valid 3 coloring of G, as desired.

Grading: 3 pts for adding a new vertex, 3 pts for adding each edge from the new vertex to the rest, 4 pts for explaining how this construction satisfies the if and only if condition of a reduction.

2) (5 pts) The same device used to do the reduction in question 1 can be used to show that 2COLOR $\leq_P$ 3COLOR. But, even if this reduction is successfully done, it does NOT prove that 2COLOR is NP-Complete. Why?

Solution

To prove that a problem is NP-Complete, you must show it belongs to the class NP, and then reduce a known NP-Complete problem to it. But in the described scenario, we are not reducing from a known NP-Complete problem. (In fact, we are actually just reducing from a problem known to be in the class P.) In some sense, the proof just shows that solving the 2COLOR problem is NO HARDER than proving the 3COLOR problem. (A polynomial solution to the latter proves that a polynomial solution to the former exists.) But, this doesn't show the difficulty of 2COLOR.

Grading: 4 pts for understanding that the order of the reduction is "reversed" in comparison to a valid proof. 1 pt for clearly tying everything together.

3) (5 pts) Let the language UNEVENPART = { <S, x, y> | S is a set of positive integers such that a subset T of S sums to x and the set S - T sums to y }

Prove that UNEVENPART is NP-Complete.

Solution

UNEVENPART belongs to NP because given a subset T, in linear time the values of T can be added and compared to x while the values not in T can be added and compared to y.

To prove UNEVENPART is NP-Complete, reduce from SUBSET-SUM in polynomial time. Since SUBSET-SUM is NP-Complete, if the reduction exists, then it follows that UNEVENPART is NP-Complete.

Given the input S and t for an instance of SUBSET-SUM, calculate S', x and y as follows:

S' = S
x = t, the target for the subset sum instance.
y = $(\sum_{z \in S} z) - t$

If S has a subset that adds up to t, then that same subset adds up to x while its complement set must add up to y, by definition.

Similarly, if S' has a set that adds up to x with a complement set that adds up to y, we only need to take the set that adds up to x, which also exists in S to find a valid subset that adds up to t in S.

Grading: 1 pt for showing in NP and choosing language from which to reduce, 4 pts for proof

4) (5 pts) In what city is Orlando International Airport?

Solution

Orlando, **give to all.**