COT 4210 Quiz #3 Part B: Undecidability, Reducibility 3/25/2021

Regular Start Time: 2:10 pm (EST)
Regular End Time: 2:45 pm (EST)
Regular Late Time: 2:55 pm (EST)

4) (10 pts) Prove that the following language is undecidable:

L_4 = { (<M>, k) | <M> is the encoding of a Turing Machine which accepts all strings of length k}

5) (12 pts) Consider the two following problems:

SAFE-CLASSES = { (S, E) | S is a list of students, and E is a list of pair of students who are not allowed to be in the same class, and there exists a way to split all students in S into two non-empty classes such that the requirements given by E are satisfied }

TWO-COLORABLE = { G | G is an undirected graph such that each vertex in G can be assigned one of two colors, Red or Blue, such that no edge in G connects two vertices that have the same color. }

SAFE-CLASSES is mapping reducible to TWO-COLORABLE. To do the reduction, given an input of students and pairs of students who can't be in the same class, you can create a graph G that is two-colorable if and only if there's a way to split the students in S into two classes satisfying the restriction.

(a) Give an unambiguous algorithmic description of how to take (S, E) and convert it to a corresponding G that completes the mapping reduction successfully.

(b) Use your mapping reduction to create a graph for the following input for the problem SAFE-CLASSES:

S = {Ariel, Binh, Carthik, Deanna, Eduardo, Freddy}
E = { (Ariel, Carthik), (Binh, Carthik), (Binh, Eduardo), (Carthik, Freddy), (Deanna, Eduardo) }

(c) Using your graph or otherwise, give a valid listing of two classes of students which proves that the input above belongs to the language SAFE-CLASSES

7) (3 pts) Who invented the idea of the Turing Machine?