**COT 4210 Quiz #3 Part A: Decidability, Coountability 3/25/2021**

**Regular Start Time: 1:35 pm (EST)**
**Regular End Time: 2:10 pm (EST)**
**Regular Late Time: 2:20 pm (EST)**

1) (6 pts) Prove that the following language, $L_1$, is a decidable language:

$L_1$ = { (<D>, k, m) | <D> is the encoding of a DFA which accepts at least k strings that are length m or less}

2) (9 pts) Prove that the following language $L_2$, is a decidable language:

$L_1$ = { (<D>, k, m) | <D> is the encoding of a DFA which accepts at least k strings that have a length greater than m}

3) (10 pts) Prove that the set of ordered pairs (x, y) where x and y are both positive integers with x < y is countable. To make sure you just don't copy a similar proof shown in class, in order to get full credit for this one, you need to do the following:

(a) Give an English description of how you would order the ordered pairs, making sure that each one gets listed exactly once. (So, based on your description, I should be able to design a computer program that starts running and spits out ordered pairs listed above without ever repeating one and such that it would eventually spit out any specified ordered pair.)

(b) Given the ordering you've described, what is the rank of the ordered pair (100, 200) in your ordering. **Please leave your answer as an arithmetic expression containing only integers and usual mathematical operations or functions ( +, -, *, /, combos and powers).**

(c) Explain why this ordering: (1, 2), (1, 3), (1, 4), … , (2, 3), (2, 4), (2, 5), …, (3, 4), (3, 5), (3, 6), … is insufficient to show that the ordered pairs described are countable.