<u>COT 4210 Final Exam Part D: Classes P/NP 5/4/2021</u>

Regular Start Time: 3:00 pm (EDT)
Regular End Time: 3:50 pm (EDT)
Regular Late Time: 4:00 pm (EDT)

1) (8 pts) A subsequence of a string s is a subset of the letters of s in the same order as they appear in s. For example, if s = PO**L**YNOMI**A**LTI**ME**, then t = **LAME** is a subsequence of s, since we can create t using the 3rd, 9th, 13th and 14th characters of s, in that order. (Characters highlighted in red.) Define the language L as follows:

L = { (s, t) | s and t are strings and t is a subsequence of the string s }.

Prove that L belongs to the class P.

2) (15 pts) In class it was shown that 3-SAT $\leq_P$ 3-COLOR. For this question, show that the polynomial time reduction can be done in the opposite order, namely that 3-COLOR $\leq_P$ 3-SAT. Note that looking at the proof that was shown in class is unlikely to help you on this question. (I am just trying to help you because I think people's natural instinct would be to look at the details of that proof, but those would not be helpful for this question and I want to save everyone the hassle of looking at that proof and losing time because of it.) Thus, here is what you must do:

(a) Start with an input, G, to the 3-COLOR problem, which is an arbitrary unweighted, undirected graph (**with no colors assigned to any vertices!!!)**

(b) Provide an algorithm to use G to create a Boolean formula φ, in 3-CNF form.

(c) Prove, that if the graph G is 3 colorable, then the corresponding Boolean formula, φ that your algorithm generated is satisfiable.

(d) Prove, that if the output formula φ is satisfiable, then then corresponding input G that created it must be 3 colorable.

(e) Prove that your algorithm to calculate φ from G runs in polynomial time of the input, G.

In the natural description of the reduction, quite a few of the clauses one would create would be the or of two variables, not three. Note that any clause of the form $(a \vee b)$ can easily be transformed into an equivalent clause with three terms as follows: $(a \vee b \vee b)$. There is no need to describe this in your reduction. Your algorithm may produce clauses of two or three terms.

3) (2 pts) What type of fruit is used to make a Pumpkin Pie?