

COT 4210 Final Exam Part C: TMs, Decidable/Undecidable Languages 5/4/2021

Regular Start Time: 2:20 pm (EDT)

Regular End Time: 3:00 pm (EDT)

Regular Late Time: 3:10 pm (EDT)

1) (8 pts) Let L_1 and L_2 be Turing Recognizable languages. Prove that $L_1 \cup L_2$ is also Turing Recognizable. In order to earn full credit, your proof must be detail oriented.

2) (8 pts) Prove that the following language C , is a decidable language:

$C = \{ \langle G \rangle, k, m \mid \langle G \rangle \text{ is a Context Free Grammar such that at least } k \text{ distinct strings are generated by applying } m \text{ or fewer rules starting from the start variable. Both } k \text{ and } m \text{ are given positive integers.} \}$

Applying a single rule means picking a single variable in the current derivation and substituting it with the right hand side of a rule for that variable. Please give your algorithm to decide membership in this language at a high level without Turing Machine level details and prove that your algorithm must terminate and return the correct result.

3) (9 pts) Let $L = \{ \langle M_1, M_2, k \rangle \mid M_1 \text{ and } M_2 \text{ are Turing Machines for which there exists a string } s \text{ such that } |s| = k \text{ and both } M_1 \text{ and } M_2 \text{ accept } s. \}$ Prove that L is undecidable from first principles (without using Rice's Theorem or any other short cut).