

**COT 4210 Final Exam Part B: Context Free Grammars 5/4/2021**

**Regular Start Time: 1:40 pm (EDT)**

**Regular End Time: 2:20 pm (EDT)**

**Regular Late Time: 2:30 pm (EDT)**

1) (5 pts) A context free grammar over the alphabet  $\{a, b\}$  with variables  $\{S, A, B, C\}$  and the start symbol  $S$  is given below:

$S \rightarrow A \mid BA$

$A \rightarrow CC \mid AC \mid \varepsilon$

$B \rightarrow CAC \mid aa \mid b$

$C \rightarrow a \mid BB$

Give a derivation for the string  $aaaabb$  in this grammar.

2) (7 pts) Design a PDA for the following language  $L$ , over the alphabet  $\{a,b,c\}$ :

$$L = \{ a^x b^y c^z \mid x, y, z \text{ are non-negative integers such that } x > y+z. \}$$

Please give either a drawing of your design or a text description with the 6-tuple of the formal definition of the PDA. If you make a drawing, recall that a transition between states labeled as

$a, b \rightarrow c$ , means that you are reading the character  $a$ , popping character  $b$  from the stack and pushing character  $c$  onto the stack.

3) (5 pts) In the last step of creating a Chomsky Normal Form grammar from any regular grammar, extra variables and extra rules are introduced so that no rule maps a variable to more than 2 variables. Show an application of this step on the following single rule:

$S \rightarrow ABCDEFG$

You may add additional variables that are English capital letters, H or later in the alphabet.

4) (8 pts) Let  $\Sigma = \{a, b, c\}$  and  $L = \{w \mid w \text{ has the same number of } a\text{'s, } b\text{'s and } c\text{'s}\}$ . Prove that  $L$  is not context free via the Pumping Lemma for Context Free Grammars.