University of Central Florida
School of Computer Science
COT 4210 Spring 2004

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Homework 7: answers

1. The following is a hierarchy of subsets of $I\!N$.

$$regular \subsetneqq CFL \subsetneqq recursive \subsetneqq re. \subsetneqq P(I\!N)$$

   Show that the hierarchy is proper by giving examples of sets contained in each level but not in the next one.

   answer: The question is poorly worded, apologies. But what I wanted was explained in class:[1]

   - $\{(2^n - 1)2^{n-1} \mid n \geq 1\}$ is context-free but not regular.
   - $\{2^{n^2} \mid n \geq 0\}$ is recursive but not context-free.
   - Let C_i be the i^{th} C program in some canonical ordering. Then $\{2^i 3^j \mid C_i$ halts on input $j\}$ is recursively enumerable but not recursive; and $\{2^i 3^j \mid C_i$ loops on input $j\}$ is not recursively enumerable.

2. In this question, let M be a Turing machine which takes as input a positive integer i and outputs (if it halts) a positive integer $M(i)$. Denote by L_M the language $\{M(i) \mid i = 1, 2, 3, \ldots\}$.

   Answer True or False (no need to justify your answer, do so at your own peril). Score is +1 for correct answer, -1 for incorrect answer.

   (a) It is always true that the language L_M is recursive. (FALSE)

   (b) The set of languages over $\{0, 1\}$ recognized by C programs is countable. (TRUE)

   (c) There exists a language recognized by M but not recognized by any Finite Automaton. (TRUE)

[1] As per convention, integers are represented in binary.

(d) Let H be the language composed of Turing machine representations which halt iff the input is an even number. Then H is computable in polynomial time. (FALSE)

(e) A simple argument for the existence of non-regular languages is that the set of DFAs is countable but the set of languages is not. However, Turing machines are allowed to have an infinite tape and therefore this argument cannot be used to prove the existence of non-recursively enumerable languages. (FALSE)

(f) The reversal of a deterministic CFL is always a deterministic CFL. (FALSE)

(g) There exists a language recognized by a PDA but not recognized by any C program. (FALSE)

(h) The complement of a recursively enumerable language is recursively enumerable. (FALSE)

(i) The set of all languages over $\Sigma = \{0, 1\}$ is countable. (FALSE)

(j) The complement of a CFL is always a CFL. (FALSE)

(k) Any language over the binary alphabet is recognized by some Turing machine. (FALSE)

(l) The complement of a recursive language is always recursive. (TRUE)

(m) Define a "bounded-stack" PDA (abreviated BS-PDA) as a PDA that halts whenever the depth of the stack is greater than some fixed bound T (note different BS-PDAs can have different bounds on the stack depth). Then every CFL can be recognized by a BS-PDA. (FALSE)

3. A "path" in a graph is a sequence of distinct adjacent vertices. Consider the language

$$L = \{(G, B) \mid B \in I\!N, \text{G a graph containing a path of length B}\}.$$

There is no known practical algorithm that decides membership in L. But suppose you find an oracle that decides membership in L. Given a graph G with n vertices:

(a) How many questions to the oracle are sufficient to find the length of a longest path in G? Explain.

answer: Binary search can be used to find the length in $\lceil \log_2 n \rceil$ queries.

(b) How many questions are sufficient to actually find a longest path? Explain.

answer: First find out the length τ of the longest path. Then, for each edge e in G ask the oracle if $G - \{e\}$ still contains a path of length τ. If yes, delete e from G. At the end, only a path of length τ will remain. The number of queries is at most $\lceil \log_2 n \rceil + \binom{n}{2}$.