University of Central Florida
School of Computer Science
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Homework 6:answers to selected problems

1. Chapter 9, exercise 1 (Sudkamp text).

2. Consider the following grammar over $\Sigma = \{a, b\}$.

   - $S \rightarrow AB + A$;
   - $A \rightarrow AS + Sb + a$;
   - $B \rightarrow BS + aA + S$.

   (i)To make $S \rightarrow A$ redundant, consider the step in a derivation where a new variable S is produced. (ii)This happens when one of the productions $A \rightarrow AS$; $A \rightarrow Sb$; $B \rightarrow BS$; $B \rightarrow S$ is used. (iii) The only place where $S \rightarrow A$ can play a role is in changing one of these newly produced S's into an A. (iv) So adding the rules $A \rightarrow AA + Ab$ and $B \rightarrow BA$ makes $S \rightarrow A$ redundant. (v) We thus obtain the equivalent grammar

   - $S \rightarrow AB$;
   - $A \rightarrow AS + Sb + a + AA + Ab$;
   - $B \rightarrow BS + aA + S + BA$.

   The above is wrong. For example, the grammar no longer generates the string "a". So the argument must be flawed. The argument consists of five statements. Find the first erroneous statement. Explain. Your answer should be precise and concise.

   answer: The third statement is false. It does not consider the case when the first rule used is $S \rightarrow A$.

3. Here is pseudocode to enumerate $\mathbb{N}X\mathbb{N}$:

```
for (plane = 0; 1; plane++)
for (i = 0; i <= plane; i++)
for (j = 0; j <= plane; j++)
if (i + j == plane) print("(i,j)");
```

Let C_i denote the i^{th} C program. We can use this ordering to, for example, run C_i on input j for all $i, j \in \mathbb{N}$.

(a) Write pseudocode to enumerate $\mathbb{N}X\mathbb{N}X\mathbb{N}$.

answer:

```
for (plane = 0; 1; plane++)
for (i = 0; i <= plane; i++)
for (j = 0; j <= plane; j++)
for (k = 0; k <= plane; k++)
if (i + j + k == plane) print("(i,j,k)");
```

(b) Assume "halts(i,j,k)" is a subroutine that runs C_i on input j for up to k steps and outputs 1 if the program halts in that time-frame (outputs 0 otherwise). Write pseudocode to enumerate the language

$$H = \{2^i 3^j \mid C_i \text{ halts on input } j\}.$$

answer:

```
for (num_steps = 0; 1; num_steps++)
for (i = 0; i <= num_steps; i++)
for (j = 0; j <= num_steps; j++)
if (i + j == num_steps)
```

if halts(i,j,num_steps) print($2^i 3^j$);