University of Central Florida
School of Computer Science
COT 4210 Spring 2004

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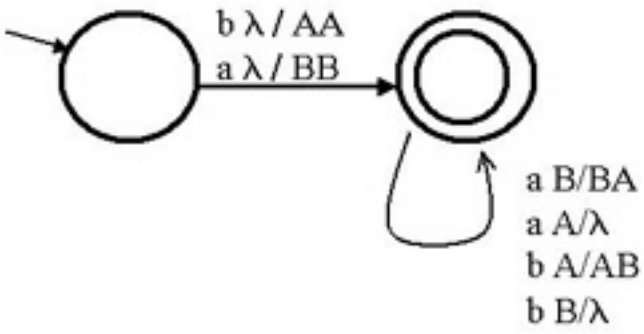
Homework 5:answers to selected problems

1. Construct an extended PDA for the language given by the following grammar

$$\begin{aligned} S &\rightarrow aBB + bAA \\ A &\rightarrow bAB + a \\ B &\rightarrow aBA + b \end{aligned}$$

answer:



2. Consider the language L generated by the following grammar.

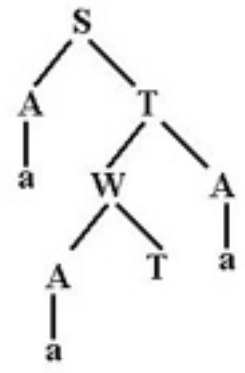
$$\begin{aligned} S &\rightarrow AT + BR + \lambda \\ W &\rightarrow AT + BR + a + b \\ T &\rightarrow WA + a \\ R &\rightarrow WB + b \\ A &\rightarrow a \\ B &\rightarrow b \end{aligned}$$

   (a) Find a minimum-length string in L which satisfies the pumping lemma for CFLs.

   answer: The idea is to find a shortest derivation tree that repeats a variable along a path from the root to a leaf. In the derivation tree below, we see that

- $S \to aaTa$
- $T \to aTa$

We also know $T \to a$. Thus the strings aa^iaa^i are in the language for all $i \geq 0$. This illustrates the general technique to solve this problem (yielding the candidate answer "aaaa"). In this particular language, however, the strings a^ia^i are in the language for all $i \geq 0$ (because of the rule $S \to \lambda$). So the answer is "aa".



(b) Find a non-empty string $u \in L$ such that $u^i \in L$ for all $i \geq 0$.

answer: $u = aa$.

(c) Find strings u, v such that $|u| \geq 6$, $v \neq \lambda$, and such that $u^iv \in L$ for all $i \geq 0$.

answer: $u = a^6 \qquad v = aa$.

The problem has more to do with basic set-theoretic operations than with formal languages.

3. (a) Find L_1, L_2 such that the symmetric difference $L_1 \oplus L_2$ is infinite and $\overline{L_1} \cap \overline{L_2}$ is not context-free.

answer: Let

- $\Sigma = \{a, b, c\}$,
- $T = \{a^nb^nc^m \mid n, m \geq 0\}$,
- $R = \{a^nb^mc^m \mid n, m \geq 0\}$.

Let $L_1 = \overline{T}$ and $L_2 = \overline{R}$. Then

- $\overline{L_1} \cap \overline{L_2} = \{a^nb^nc^n \mid n \geq 0\}$; and
- $\{a^nb^n \mid n \geq 0\} \subset L_1 \oplus L_2$.

(b) Find non-regular CFLs L_1, L_2 such that the symmetric difference $L_1 \oplus L_2$ is infinite and $\overline{L_1} \cap \overline{L_2}$ is context-free. **answer:** There are many possible examples. Here is one. Let

- $\Sigma = \{a, b\}$,
- $L_1 = \{a^{2n}b^{2n} \mid n \geq 0\}$,
- $L_2 = \{a^{2n+1}b^{2n+1} \mid n \geq 0\}$.

Then

- $L_1 \oplus L_2 = \{a^n b^n \mid n \geq 0\}$;
- $\overline{L_1} \cap \overline{L_2} = \overline{L_1 \cup L_2} = \overline{\{a^n b^n \mid n \geq 0\}}$, which is context-free because $\{a^n b^n \mid n \geq 0\}$ is deterministic context-free.

4. Consider the type of automaton that you obtain by replacing the stack in a PDA by a queue. Let us call this type of machine a FIFO-automaton. For simplicity, assume FIFO-automata accept by accepting state only.

   - Briefly explain how a FIFO automaton can implement "PUSH(X,Q)" (add X to the front of Q), "TAIL(Q)" (read the tail element in Q), and "REMOVE-TAIL(Q)" (remove the tail element in Q).
   - Does there exist a non-context free language L that is accepted by a FIFO-automaton? Justify your answer.

   answer: we covered this ad-nauseum in class.