University of Central Florida
School of Computer Science
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Extra credit HW: answers to selected problems

1. Consider the following grammar over $\Sigma = \{a, b\}$:

   - $S \to AB$;
   - $A \to AS + Sb + a$;
   - $B \to BS + aA + \lambda$;

   Convert the grammar to Chomsky Normal Form (please use rules $U \to a$ and $V \to b$). **answer:** Besides some simpler transformations, we must remove unallowed λ-productions and chain rules. The general technique I taught in class is

   > to remove a given production P, add redundant productions until P itself becomes redundant. Then remove P.

   First deal with the mixed (terminals and variables) terms using $U \to a$ and $V \to b$:

   - $S \to AB$;
   - $A \to AS + SV + a$;
   - $B \to BS + UA + \lambda$;

   To make $B \to \lambda$ redundant, add $S \to A$ and $B \to S$:

   - $S \to AB + A$;
   - $A \to AS + SV + a$;
   - $B \to BS + UA + S$.

   To make $S \to A$ redundant, add $S \to AS + SV + a$. Reasoning: when the rule $S \to A$ is used in a derivation, the variable A must at some point later be substituted for AS or SV or a.

- $S \to AB + AS + SV + a$;
- $A \to AS + SV + a$;
- $B \to BS + UA + S$.

Now we could remove $B \to S$ in the same way we removed $S \to A$. Instead, and to show you that there is more than one way to do this, consider the step in a derivation where a new variable B is produced. This can only happen when the production $S \to AB$ is used.[1] The only place where $B \to S$ can later play a role is in changing this B into an S. So adding the rule $A \to AS$ makes $B \to S$ redundant. But this rule is already there, so we can simply remove the rule $B \to S$:

- $S \to AB + AS + SV + a$;
- $A \to AS + SV + a$;
- $B \to BS + UA$.

Eliminating recursion on S is simple. The final grammar is

- $S \to AB + AT + TV + a$;
- $T \to AB + AT + TV + a$;
- $A \to AT + TV + a$;
- $B \to BT + UA$;
- $U \to a$;
- $V \to b$.

[1] Why can we ignore the production $B \to BS$ here?

2. Consider the following grammar over $\Sigma = \{a, b\}$:

   - $S \to SAS + a$;
   - $A \to AS + b$;

   Convert the grammar to Greibach Normal Form. Show and comment your work.

   answer: We must eliminate left-recursion. We note left-derivations from S generate the following expressions

   $$S \to a(AS)^* = a + a(AS)^+.$$

   We can do this without left-recursion as follows:

   - $S \to a + aT$;
   - $T \to AST + AS$.

   Similarly, A generates the following expressions

   $$A \to bS^* = b + bS^+.$$

   We can do this without left-recursion as follows:

   - $A \to b + bR$;
   - $R \to SR + S$.

   The resulting grammar is

   - $S \to a + aT$;
   - $T \to AST + AS$;
   - $A \to b + bR$;
   - $R \to SR + S$.

Greibach Normal Form requires expressions on the right-hand side of productions to begin with non-terminals. Simple substitution can be used to obtain

- $S \rightarrow a + aT$;
- $T \rightarrow (b + bR)ST + (b + bR)S$;
- $A \rightarrow b + bR$;
- $R \rightarrow (a + aT)R + a + aT$.

The text does not allow S in the right-hand side of productions in Greibach Normal Form. However, it was noted in class that there appears to be no good reason for this requirement. So the above is good enough.