WEEK 9

CFLs

CLOSURE

EASY:

CONCATENATION: $L_A \cdot L_B$

G'_{CAT} $S \to S_A S_B$ ← START FOR L_B

START FOR L_A

KLEENE STAR: L_A^*

G'_{STAR} $S \to S_A S \mid \lambda$

START FOR L_A

NOTE: $G_A = (V_A, \Sigma, R_A, S_A)$

$G_B = (V_B, \Sigma, R_B, S_B$

$G_{CAT} = (\{S\} \cup V_A \cup V_B, \Sigma, \{S \to S_A S_B\} \cup R_A \cup R_B, S)$

$G_{STAR} = (\{S\} \cup V_A, \Sigma, \{S \to S_A S \mid \lambda\} \cup R_A, S)$

CFL CLOSURE

EASY: UNION

$G_A = (V_A, \Sigma, R_A, S_A)$
$G_B = (V_B, \Sigma, R_B, S_B)$
$G = (\{S\} \cup V_A \cup V_B, \Sigma, \{S \to S_A | S_B\} \cup R_A \cup R_B, S]$

MODERATE: INTERSECTION WITH REGULAR

$Q_0 = (Q_0, \Sigma, \Gamma, \delta_0, q_0, \$, F_0)$ PDA, $L_0 = L(Q_0$
$Q_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ DFA $= L_1 = L(Q$

DEFINE $Q_2 = (Q_0 \times Q_1, \Sigma, \Gamma, \delta_2, \langle q_0, q_1 \rangle, \$, F_0 \times F_1)$

$\delta_2(\langle q, s \rangle, a, X) \supseteq \{(\langle q', s' \rangle, \alpha)\}$, $X \in \Gamma$, $a \in \Sigma_e$

IFF $\delta_0(q, a, X) \supseteq \{(q', \alpha)\}$

AND $\delta_1(s, a) = s'$ IF $a \in \Sigma$
$s' = s$ IF $a = \lambda$ TREAT AS NO STATE $s' = s$
ELSE $\delta_1(s, a) = s$

NOW INDUCTIVELY CAN SHOW

$[\langle q_0, q_1 \rangle, w, \$] \vdash^* [\langle t, s \rangle, \lambda, B]$ IFF
$[q_0, w, \$] \vdash^* [t, \lambda, B]$ IN Q_0
AND $[q_1, w] \vdash^* [s, \lambda]$ IN Q_1

SO $w \in F(Q_2)$ IFF $t \in F_0$ & $s \in F_1$ IFF $w \in F(Q_0) \cap F(Q_1)$

CFL CLOSURE

MODERATE: SUBSTITUTION

$G = (V, \Sigma, R, S)$ $L = \mathcal{L}(G)$

SUBSTITUTION

$f(a) = L_a$ $a \in \Sigma$ L_a A CFL

$G = (V_a, \Sigma, R_a, S_a)$ $L_a = \mathcal{L}(G_a)$

ASSUME SAME OR USE Σ_a

IN R, CHANGE ALL INSTANCES OF $a \in \Sigma$ IN RHS's TO S_a

THUS, IF ORIGINALLY,

$S \Rightarrow a_1, \ldots, a_k$

THEN, IN NEW

$S \overset{*}{\Rightarrow} S_{a_1} \cdots S_{a_k}$

AND THEN

$S \overset{*}{\Rightarrow} w_{a_1} \cdots w_{a_k}$

WHERE $w_{a_i} \in f(a_i)$

$G' = (V \cup V_A \cup V_B, \Sigma, R', S)$ MAY NEED $\bigcup_{a \in \Sigma} \Sigma_a$

$R' = R^{CHANGED} \cup R_a \; a \in \Sigma$

WHERE $R^{CHANGED}$ IS AS ABOVE

WEEK 9
CFL NON-CLOSURE

INTERSECTION

BY EXAMPLE OF CONTRADICTORY CASE

$L_1 = \{a^n b^n c^m \mid n, m > 0\}$
$L_2 = \{a^m b^n c^n \mid n, m > 0\}$

$S_1 \to S_1 c \mid T_1 c$ $\quad$ $S_2 \to a S_2 \mid a T_2$
$T_1 \to a T_1 b \mid a b$ $\quad$ $T_2 \to b T_2 c \mid b c$

BOTH ARE CFLs

HOWEVER,
$L_1 \cap L_2 = \{a^n b^n c^n \mid n > 0\}$
WHICH IS NOT A CFL

COMPLEMENT

BY FACT THAT CLOSURE UNDER UNION AN COMPLEMENT IMPLIES CLOSURE UNDER INTERSECTION

$\sim(\sim A \cup \sim B) = A \cap B$

COMPLEMENT EXAMPLE FOR CFL

$L = \{ww \mid w \in \{a,b\}^+\}$
IS NOT A CFL BUT

$\bar{L} = \{z \mid |z| \text{ IS ODD AND } z \in \{a,b\}^+\}$
$\cup \{XY \mid |X| = |Y| \text{ AND } X \neq Y\}$

FIRST PART IS REGULAR
SECOND PART IS SEEN AS

$X_1 a X_2 Y_1 b Y_2$ OR $X_1 b X_2 Y_1 a Y_2$

WHERE $|X_1| = |Y_1|$, $|X_2| = |Y_2|$
BUT X_1, X_2, Y_1, Y_2 VALUES ARE UNIMPORTANT

SO LOOK AT AS
$X_1 a \underbrace{Y_1 X_2}_{\text{LENGTH ONLY}} b Y_2$ AND $X_1 b \underbrace{Y_1 X_2}_{\text{LENGTH ONLY}} b Y_2$

$S \to AB \mid BA$
$A \to XAX \mid a$
$B \to XBX \mid b$
$X \to a \mid b$

LOTS OF CLOSURE

SUBSTITUTION & INTERSECTION w/REGULAR

$f(a) = \{a, a'\}$

$g(a) = \{a'\}$

$h(a) = \{a\}; \ h(a') = \{\lambda\}$

PREFIX

INFIX

SUFFIX

QUOTIENT WITH REGULAR

ETC.

$L/R = h(f(L) \cap (\Sigma^* \cdot g(R)))$

$x \cdot y'$ $\quad x \in \Sigma^*$, $y \in R$

$x \cdot y'$ $\quad xy \in L$, $y \in R$

BUT <u>NOT</u>

QUOTIENT WITH CFL

MIN AND MAX

$MAX(L_1) = \{x \mid x \in L \text{ AND } xy \notin L \text{ IF } |y| > 0\}$

$MIN(L_2) = \{x \mid x \in L \text{ AND NO PROPER PREFIX OF } x \in L\}$

$L_1 = \{a^i b^j c^k \mid k \le i \text{ OR } k \le j\}$

$MAX(L_1) = \{a^i b^j c^k \mid k = MAX(i,j)\}$ NOT CFL

$MIN(L_1) = \{\lambda\}$ REGULAR

$L_2 = \{a^i b^j c^k \mid k > i \text{ OR } k > j\}$

$MAX(L_2) = \{\}$ REGULAR

$MIN(L_2) = \{a^i b^j c^k \mid k = MIN(i,j) + 1\}$ NON-CFL

BOTH L_1 AND L_2 ARE CFLs

NON-CLOSURE HERE

WEEK 9

SOLVABLE PROBLEMS
CFLs

$w \in L$?	RUN CKY WITH CNF, G IF S IN FINAL CELL $w \in L$
$L = \emptyset$	REDUCE G IF S NON-PRODUCTIVE $L = \emptyset$; ELSE $L \neq \emptyset$
L FINITE	REDUCE G RUN DFS (S) IF NO LOOPS, L FINITE; ELSE L INFINITE

10/20/2016 WEEK 9 CSG

$\alpha \to \beta$ $\quad |\alpha| \le |\beta|,$

$\alpha \in (V \cup \Sigma)^* V (V \cup \Sigma)^*$

$\beta \in (V \cup \Sigma)^+$

ONE EXCEPTION IS

$S \to \lambda$ IF $\lambda \in L$

AND THEN S CANNOT BE ON RHS

$L = \{a^n b^n c^n \mid n > 0\}$

$G = (\{A, B, C\}, \{a, b, c\}, R, A)$

$A \to aBbc \mid abc$

$B \to aBbC \mid abC$

$Cb \to bC$

$Cc \to cc$

$010 \ldots\ldots \mid <0>$, $0<1>$, $<0>$

$L = \{ww \mid w \in \{0,1\}^+\}$

$S \to 00 \mid 11 \mid 0A<0> \mid 1A<1>$

$A \to 0AZ \mid 1AX \mid 0Z \mid 1X$

$\left.\begin{array}{l} Z0 \to 0Z \\ Z1 \to 1Z \\ X0 \to 0X \\ X1 \to 1X \end{array}\right\}$ SHUTTLE

$Z<0> \to 0<0>$

$Z<1> \to 1<0>$

$X<0> \to 0<1>$

$X<1> \to 1<1>$

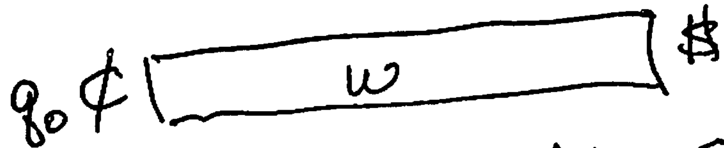
$<0> \to 0$

$<1> \to 1$

10/20/2016

WEEK 9
LBA

SIMPLE VIEW
R/W TAPE



ACCEPT BY FINAL STATE

ACTIONS ARE
READ WRITE MOVE (LEFT/RIGHT/STAY)

OFTEN ~~EASIEST~~ TO VIEW
OPERATIONS AS BEING ABLE
TO LOOK LEFT OR RIGHT (BASICALLY
MOVES EITHER WAY)

CAN ALSO VIEW AS MULTITRACK
(FINITE # OF TRACKS)
FOR EXAMPLE $(\{¢,\$\} \cup \Sigma) \times (\{¢,\$\} \cup \Gamma)$
AS TAPE ALPHABET WITH CHANNEL
1 HAVING INPUT.

QUICK EXAMPLE OF LBA

$L = \{a^n b^n c^n \mid n > 0\}$

$q_0 ¢ \rightarrow ¢ q_1$

$q_1 a \rightarrow x q_2$

$q_2 b \rightarrow y q_3$

$q_3 c \rightarrow q_4 z$

$q_2 a \rightarrow a q_2$

$q_3 b \rightarrow b q_3$

$z q_4 \rightarrow q_4 z$

$q_2 y \rightarrow y q_2$

$q_3 z \rightarrow z q_3$

$b q_4 \rightarrow q_4 b$

$y q_4 \rightarrow q_4 y$

$a q_4 \rightarrow q_4 a$

$x q_4 \rightarrow x q_1$

$q_f z \rightarrow z q_f$

$q_1 y \rightarrow y q_f$

$q_f y \rightarrow y q_f$

$q_f \$ \rightarrow \$ q_f$

$q_0 ¢ a^n b^n c^n \$ \vdash^* ¢ x^k q_1 a^{n-k} y^k b^{n-k} z^k c^{n-k} \$$

$\vdash^* ¢ x^n q_1 y^n z^n \$ \vdash^* ¢ x^n y^n z^n \$ q_f$

TAPE ALPHABET IS

$\{¢, \$, x, y, z, a, b, c\}$

0/20/2016 WEEK 9 (13)

ALGORITHMS AND PROCEDURES

PROCEDURES ARE JUST PROGRAMS OR PROCESSES THAT ARE CLEARLY COMPUTABLE.

PROCEDURES NEED NOT HALT FOR ALL INPUT

ALGORITHMS ARE PROCEDURES THAT HALT ON ALL INPUT

PREDICATES ARE PROCEDURES THAT PRODUCE ANSWERS (OUTPUT) TRUE/FALSE (YES/NO, 1/0)

DECISION PROBLEMS ARE PROBLEMS WHERE EACH INSTANCE HAS A TRUE/FALSE ANSWER

NOTATION:

CONVERGES

$f(x)\downarrow$ MEANS PROCEDURE f HALTS (GIVES AN OUTPUT) WHEN EVALUATED AT x

$f(x)\uparrow$ MEANS PROCEDURE f DIVERGES WHEN EVALUATED AT x

IF f IS AN ALGORITHM THEN $\forall x\, f(x)\downarrow$

7/20/2016

WEEK 9
COMPUTABILITY

SOLVED IS CONCRETE

SOLVABLE IS EXISTENTIAL

E.G., P=NP? IS SOLVABLE

ANSWER IS "YES" OR "NO"
AND ALGORITHMS EXIST FOR
EACH POSSIBLE ANSWER.

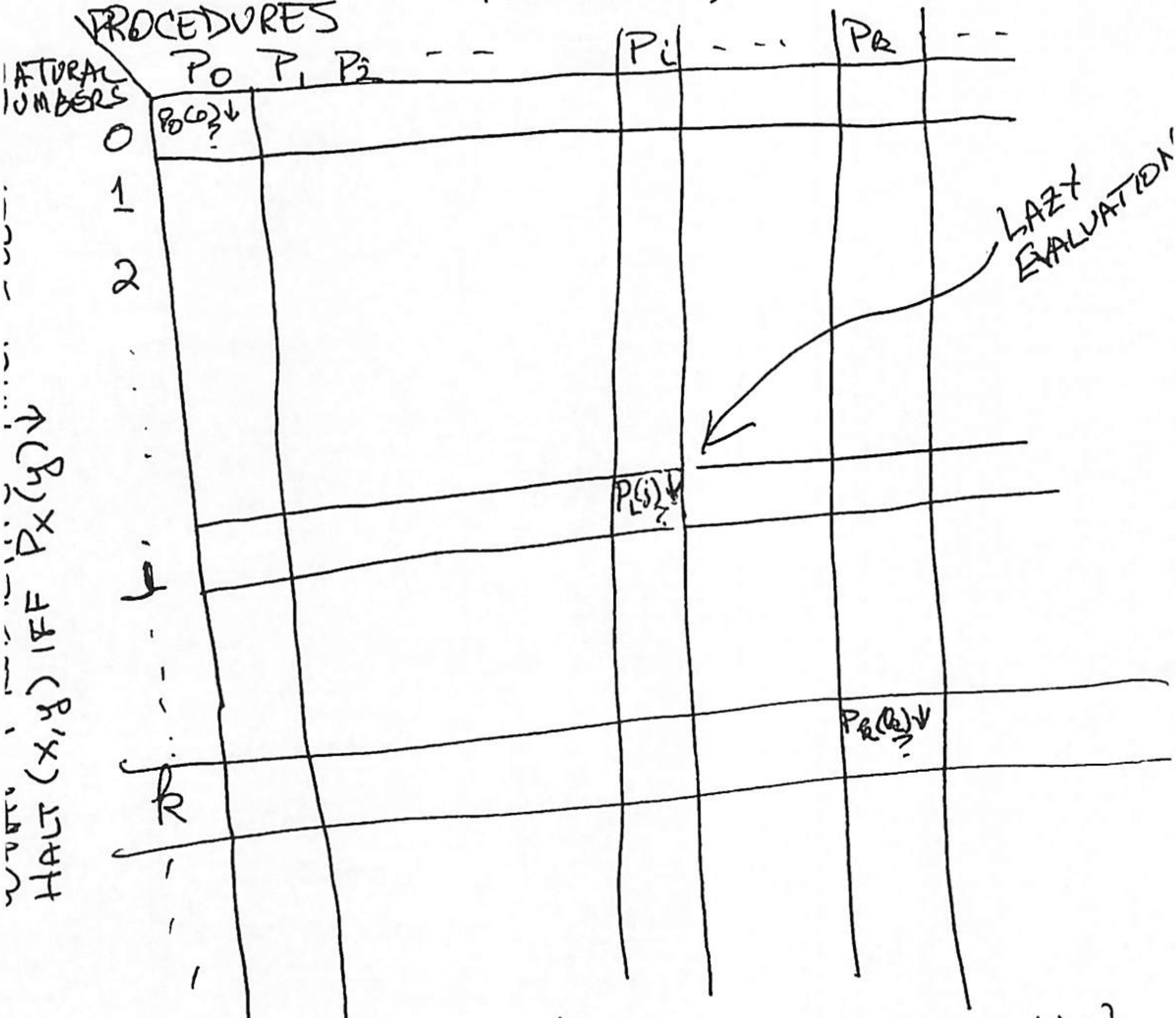
P=NP? IS UNSOLVED AS NO
ONE HAS PROVED THE PROPERTY
TO BE SO, OR HAS REFUTED IT.

$$\nexists\ a, b, c\ [a^n = b^n + c^n]$$
$$a, b, c \in \mathbb{Z}^+,\ n > 2$$

~~2/20/2016~~ WEEK 9

HALTING PROBLEM

ASSUME WE CAN DECIDE FOR ARBITRARY PROCEDURE, P, AND INPUT X, WHETHER OR NOT $P(X)\downarrow$

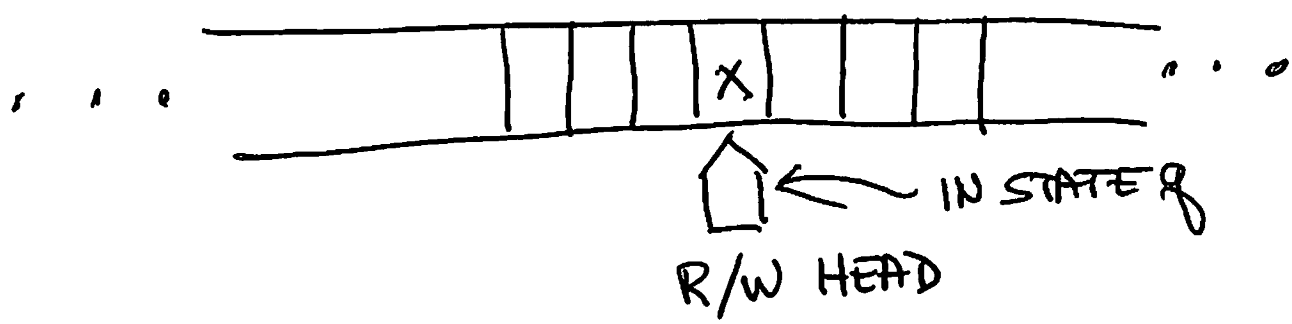


$$\text{DEFINE } D(x) = \{\text{WHILE } P_x(x);\ \text{RETURN}(1);\}$$
$$= \{\text{WHILE HALT}(x,x);\ \text{RETURN}(1);\}$$

SINCE HALT IS AN ALGORITHM, IT IS A PROCEDURE AND HENCE SO IS D. ASSUME $D = P_d$ THEN

$D(d)\downarrow$ IFF $P_d(d)\downarrow$ IFF HALT(d,d)=TRUE IFF $P_d(d)\uparrow$ IFF $D(d)\uparrow$

WEEK 9
TURING MACHINE



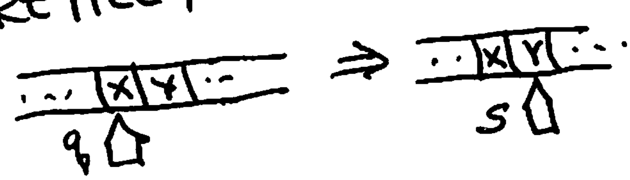
CAN READ CHARACTER IN SCANNED CELL (SAY x) AND BASED ON CURRENT STATE (SAY q) AND x (q, x IS CALLED THE DISCIMINANT)

CAN EITHER
- (a) REWRITE x AS SOME OTHER SYMBOL
- (b) MOVE RIGHT ; OR
- (c) MOVE LEFT

AND THEN CHANGE STATE

NOTE: THIS IS REALLY POST'S NOTATION

q x R s



~~10/20/2016~~ WEEK 9 16

TM TAPE

UNMARKED PARTS OF TAPE ARE BLANK
TAPE STARTS WITH INPUT (FINITE)
AT EACH STAGE IT MIGHT MOVE TO
PARTS OF TAPE NEVER VISITED BEFORE
AND MAY WRITE NEW VALUES.

BASED ON THIS, TAPE IS ALWAYS
FINITELY MARKED AND ONLY A FINITE
NUMBER OF CELLS CAN BE VISITED
IN ANY FINITE PERIOD OF TIME

FOR OUR PURPOSES, WE WILL USE
TAPE ALPHABET OF $\{0, 1\}$ AND

BLANK

DENOTE NUMBERS IN UNARY.

$M = (Q, \{0,1\}, T)$ TABLE OF QUAD
MAPPING
$Q \times \{0,1\} \to Q \times \{0,1,R,L\}$

HALTING IS JUST ENTERING A STATE
q WITH INPUT x, WHERE qx HAS NO
ENTRY IN T.