WEEK 8
PUSHDOWN AUTOMATA
PDA

$$A = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

- Q: STATES (FINITE)
- Σ: LANG. ALPHABET (FINITE)
- Γ: STACK SYMBOLS (FINITE
- δ: TRANSITION FUNCTION
- q_0: START STATE
- Z_0: OPTIONAL BOTTOM OF STACKMARKER
- F: OPTIONAL FINAL STATES

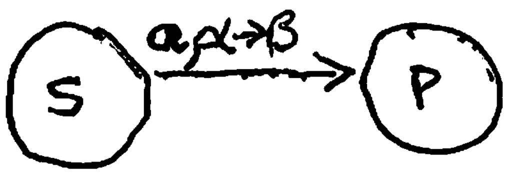
TRANSITION FUNCTION

$$\delta : Q \times \Sigma_e \times \Gamma_e \rightarrow 2^{Q \times \Gamma^*}$$

CAN EXTEND TO Γ^* CAN LIMIT TO Γ_e BY GROWING STATES

PDA DIAGRAMS

SOME TEXTS USE $a, X / Y$ WHICH IS FINE ALSO



$$\delta(s, a, \alpha) \supseteq \{(p, \beta)\}$$

PDA LANGUAGES
$\equiv$ CFLs

INSTANTANEOUS DESCRIPTIONS (ID)

$[q, w, \gamma]$ q - CURRENT STATE
w - REMAINIG INPUT
γ - STACK CONTENTS
READ LEFT (TOP) TO RIGHT (BOTTOM)

SINGLE STEP
$[q, ax, Z\alpha] \vdash [p, x, \beta\alpha]$ IF $\delta(q, a, Z) \in (p, \beta)$

MULTISTEP $\vdash^*$ REFLEXIVE TRANSITIVE CLOSURE OF $\vdash$

GIVEN $A = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$
THERE CAN BE THREE NOTIONS OF ACCEPTANCE

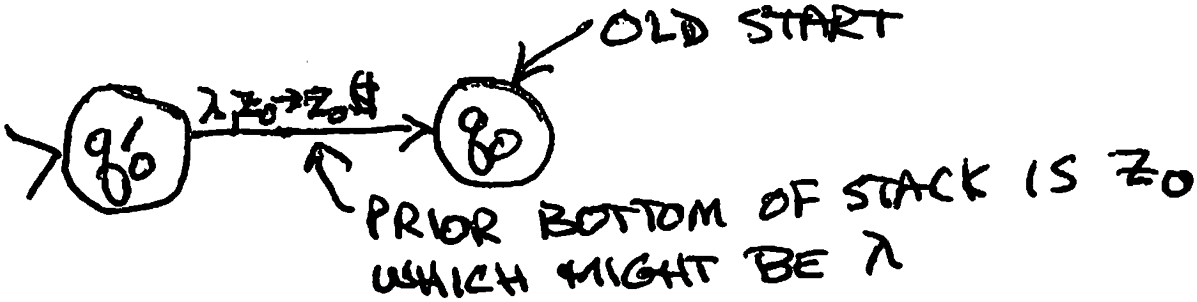
FINAL STATE
$\mathcal{L}(A) = \{w \mid [q_0, w, Z_0] \vdash^* [f, \lambda, \beta]\}$, $f \in F$

EMPTY STACK
$N(A) = \{w \mid [q_0, w, Z_0] \vdash^* [q, \lambda, \lambda]\}$, $q \in Q$

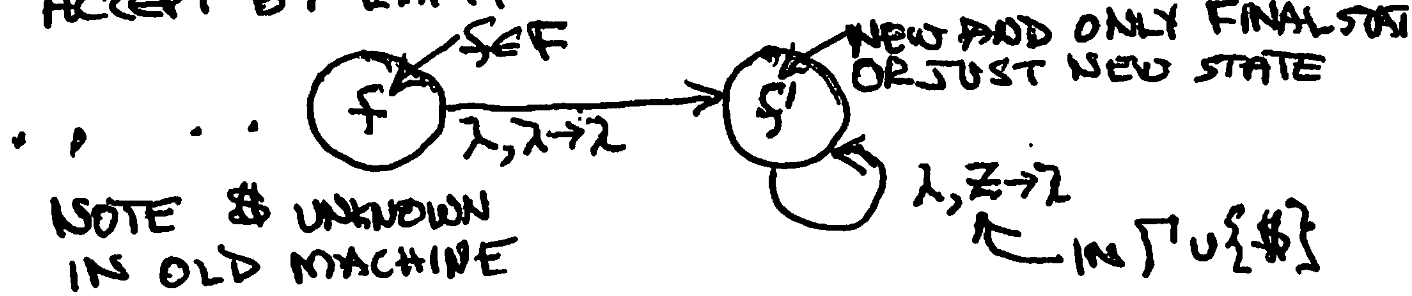
EMPTY STACK AND FINAL STATE
$E(A) = \{w \mid [q_0, w, Z_0] \vdash^* [f, \lambda, \lambda]\}$, $f \in F$

EQUIVALENCY OF LANGUAGE CLASSES, $\mathcal{L}(A)$, $N(A)$, $E(A)$, WHERE A RANGES OVER ALL PDAs

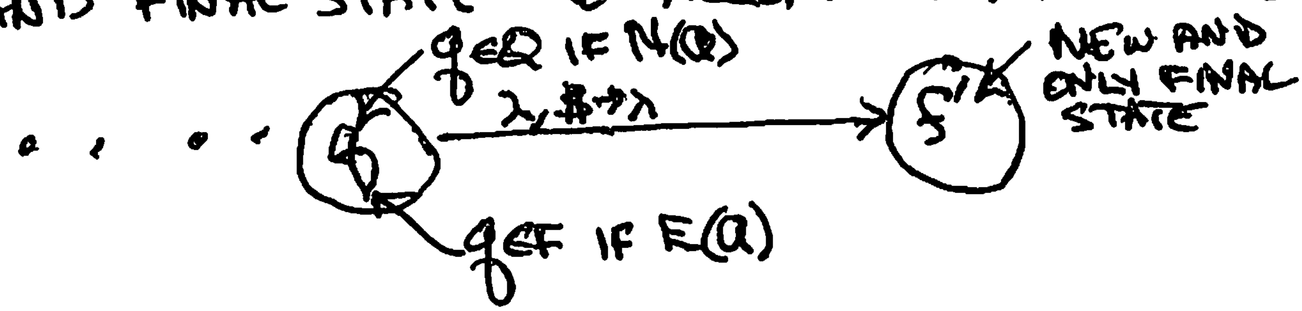
- CONVERTING ONE FORM TO ANOTHER
  FOR EACH CASE, ASSUME q_0' IS A NEW STATE (NEW START) AND \$ IS A NEW STACK SYMBOL PLACED ON BOTTOM OF STACK. FOR ALL CASES, START WITH



- CHANGE ACCEPT BY FINAL STATE TO ACCEPT BY EMPTY STACK OR EMPTY AND FINAL STATE



- CHANGE ACCEPT BY EMPTY STACK OR EMPTY AND FINAL STATE TO ACCEPT BY FINAL STATE



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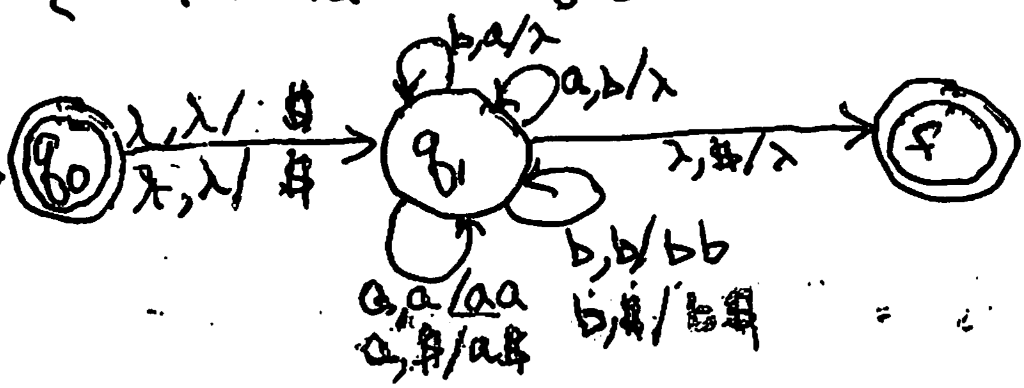
EXAMPLE PDA

$L_1 = \{ w \mid |w|_a = |w|_b \}$. ASSUME λ ON STACK

Q_1:



I USED / WHERE BOOK USES →

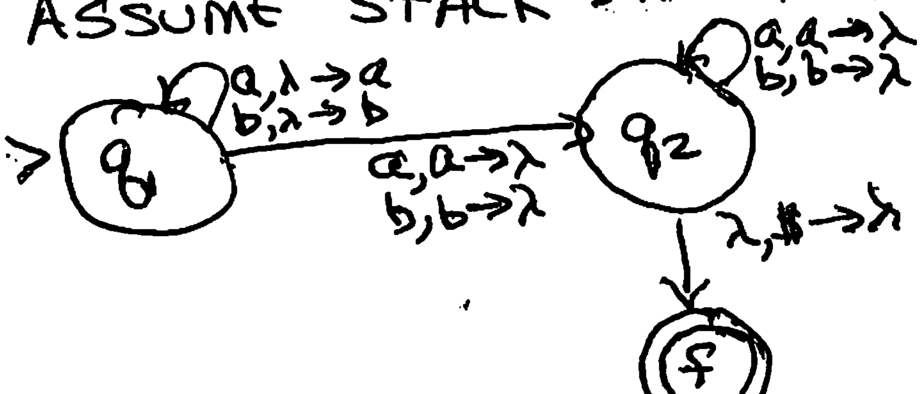
THIS GETS $L_1 = E(Q_1)$

THIS IS NON-DETERMINISTIC

$L_2 = \{ w w^R \mid w \in \{a, b\}^+ \}$

ASSUME STACK STARTS WITH \$

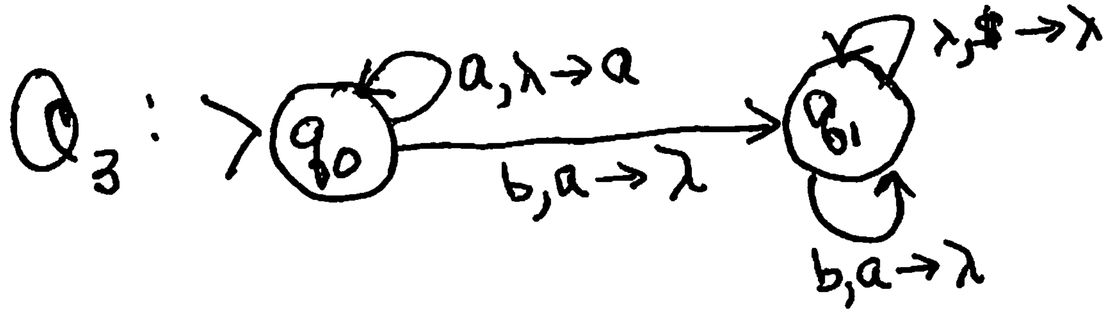
Q_2:



THIS GETS $L_2 = \mathcal{L}(Q_2) = E(Q_2)$

THIS IS ALSO NON-DETERMINISTIC

$L_3 = \{a^n b^n \mid n > 0\}$. ASSUME \$ ON STACK



THIS GETS $L_3 = N(Q_3)$

THIS IS DETERMINSTIC

DETERMINISM FOR PDAs

1) FOR EACH $q \in Q$ & $z \in \Gamma$ & $a \in \Sigma$
   IF $|\delta(q, \lambda, z)| > 0$ THEN $|\delta(q, a, z)| = 0$

2) FOR NO $q \in Q$, $z \in \Gamma$ & $a \in \Sigma_\varepsilon$
   IS $|\delta(q, a, z)| > 1$

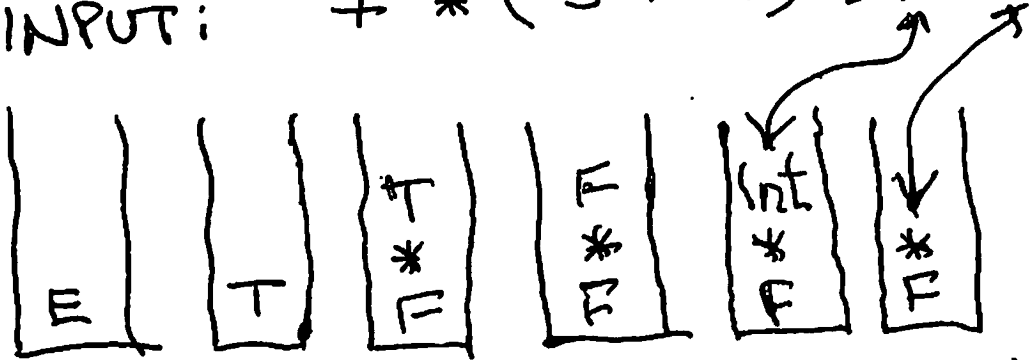
TOP-DOWN PARSER

$E \to E+T \mid T$

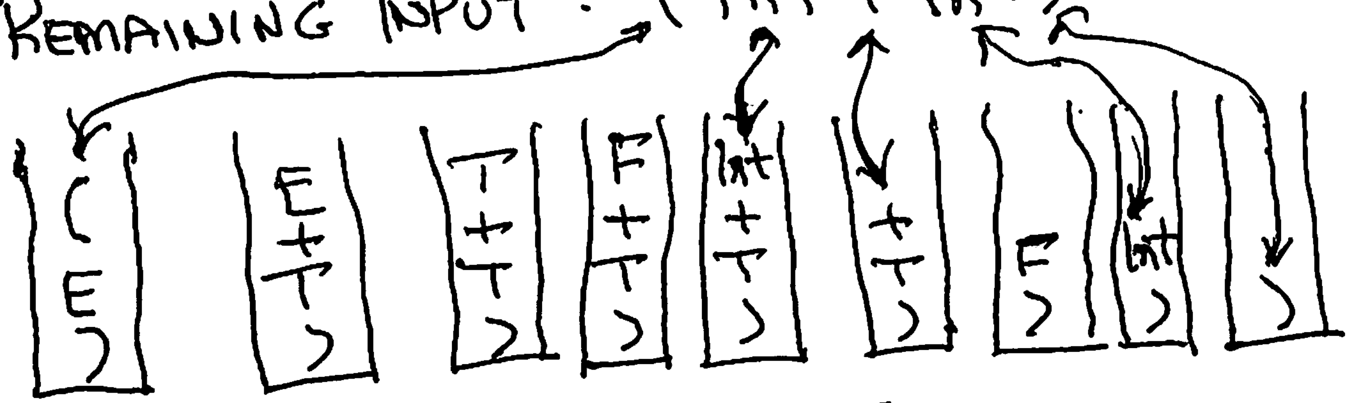
$T \to T*F \mid F$

$F \to (E) \mid int$

INPUT: $7 * (3+21) \Rightarrow int * (int+int)$



REMAINING INPUT: (Int + Int)

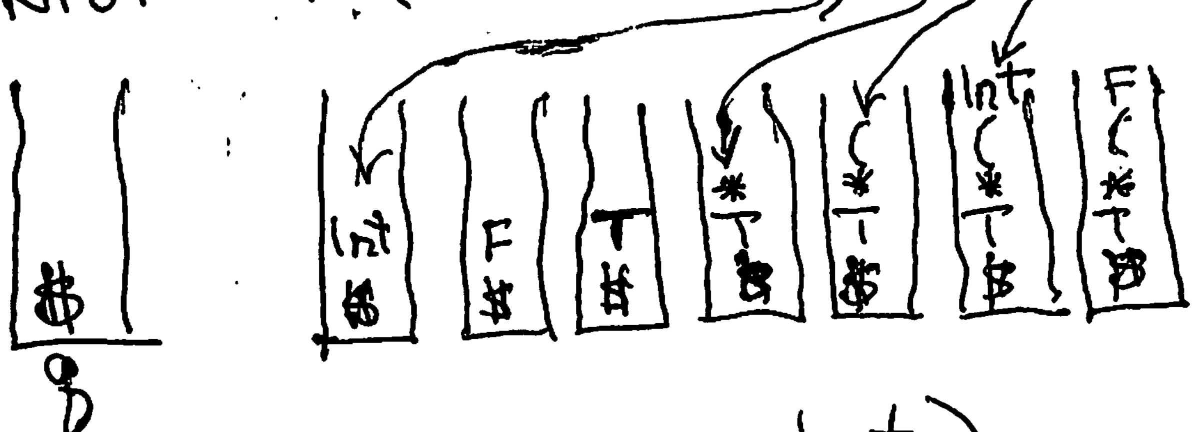


ACCEPTS BY EMPTY STACK

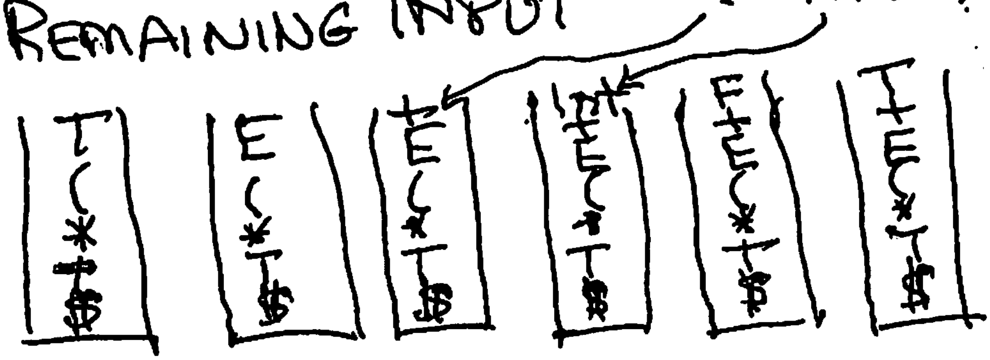
BOTTOM-UP PARSER

E → E+T | T
T → T*F | F
F → (E) | Int

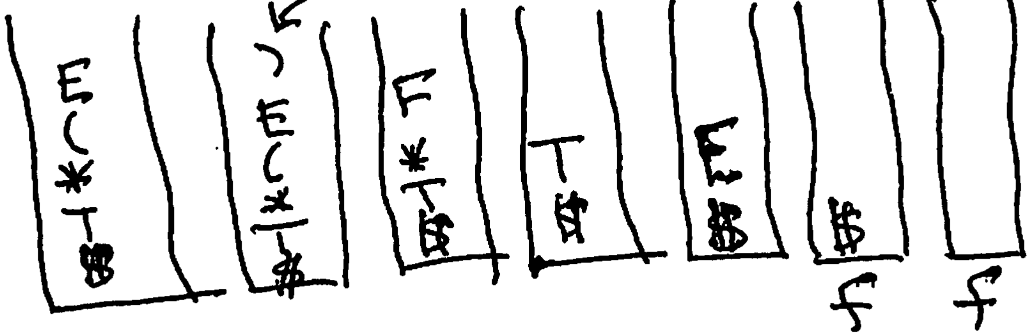
INPUT: 7*(3+21) ⇒ Int*(Int+Int)



REMAINING INPUT + Int)



REMAINING INPUT)



ACCEPTS BY FINAL STATE AND EMPTY STATE
OR JUST EMPTY STACK (CAN ALTER FOR FINAL)

LIMITING PDA TO PUSH/POP

PUSH: PUSH(α) IS EQUIVALENT TO

$$\delta(q, a, z) \supseteq \{(p, \alpha z)\}$$

WHERE WE JUST USE

$$\delta(q, a, z) \supseteq \{(p, \text{PUSH}(\alpha))\}$$

POP: POP IS EQUIVALENT TO

$$\delta(q, a, z) \supseteq \{(p, \lambda)\}$$

WHERE WE JUST USE

$$\delta(q, a, z) \supseteq \{(p, \text{POP})\}$$

IF WANT TO SIMULATE STANDARD OPERATION OF

$$\delta(q, a, z) \supseteq \{(p, \alpha)\}$$

CAN DO

$$\delta(q, a, z) \supseteq \{(p', \text{POP})\}$$

$$\delta(p', \lambda, X) \supseteq \{(p, \text{PUSH}(\alpha))\}$$

ANY ELEMENT OF Γ OR λ IF ALLOWED

$$[q_0, w, \$] \vdash^* [f, \lambda, \lambda]$$

PDA TO CFG

$$Q = (Q, \Sigma, \Gamma, \delta, q_0, \emptyset, \{f\})$$

TECHNIQUE # 1:

LIMITED TO PUSH/POP

NON-TERMINALS ARE OF FORM

$A_{t,u}$

GOAL IS $A_{t,u} \overset{*}{\Rightarrow} w \in \Sigma^*$

WHEN $[t, w, \alpha] \vdash^* [u, \lambda, \alpha]$

START IS

$A_{q_0, f}$

RULES ARE

$A_{q,q} \to \lambda$ $\quad \forall q \in Q$ REFLEXIVE

$A_{t,u} \to A_{t,v} A_{v,u}$ $\quad$ TRANSITIVE

AND

$A_{t,u} \to a A_{r,s} b$

WHEN $\delta(t, a, \lambda) \supseteq \{(r, x)\}$ PUSH

& $\delta(s, b, x) \supseteq \{(u, \lambda)\}$ POP

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PDA TO CFG

$\mathcal{Q} = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$

F → {f}

z → USED IN TECHNIQUE 1 Z=$; NOT USED IN BOOK'S APPROACH SO Ø

δ → LIMITED TO PUSH + POP

TECHNIQUE #2:

NON-TERMINALS ARE OF FORM

$\langle p, z, q\rangle$

GOAL IS $\langle p, z, q\rangle \overset{*}{\Rightarrow} w \in \Sigma^*$

WHEN $[p, w, z] \vdash^* [q, \lambda, \lambda]$

START SYMBOL IS

$S \rightarrow \langle q_0, \$, f\rangle$

CAN ACTUALLY DO FOR EMPTY STACK ONLY BY HAVING

$S \rightarrow \langle q_0, \$, q\rangle \quad \forall q \in Q$

RULES, OTHER THAN START, ARE

$\langle q, x, p\rangle \rightarrow a\langle s, Y, t\rangle\langle t, x, p\rangle$

WHENEVER $\delta(q, a, x) \supseteq \{(s, PUSH(Y))\}$

$\langle q, X, p\rangle \rightarrow a$

WHENEVER $\delta(q, a, X) \supseteq \{(p, POP)\}$

GOAL: $\langle q_0, \$, f\rangle \overset{*}{\Rightarrow} w$, WHENEVER $w \in L(\mathcal{Q})$

GREIBACH NORMAL FORM

ALL RULES, EXCEPT PERHAPS, $S \to \lambda$

LIMITED TO

$$A \to a\alpha \qquad A \in V, a \in \Sigma, \alpha \in V^*$$

PROVIDES LINEAR PARSE IF

WE CAN AVOID

SHIFT/REDUCE

&

REDUCE/REDUCE

CONFLICTS