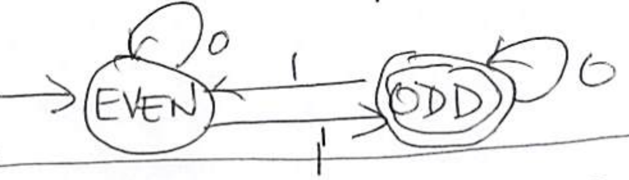
(1)

WEEK #2
SAMPLES

ODD PARITY $\Sigma = \{0,1\}$ $Q = \{EVEN, ODD\}$
$F = \{ODD\}$ $q_0 = EVEN$

$Q_1 \rightarrow$


CHECK FOR 2'S COMPLEMENT

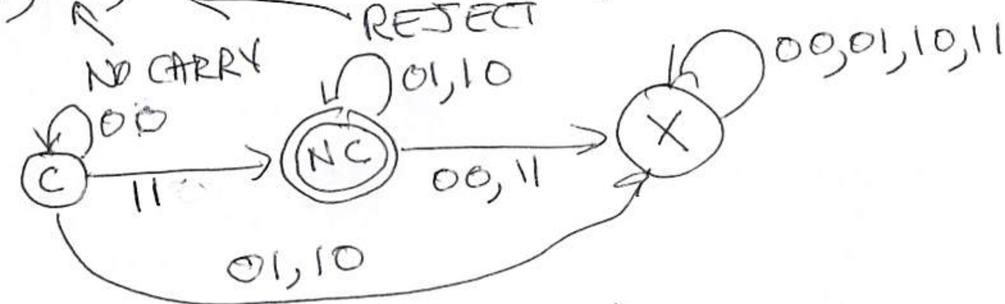
$\Sigma = \{0,1\} \times \{0,1\} = \{00, 01, 10, 11\}$

THINK OF THIS AS TWO SYCHRONIZED CHANNELS OF INPUT (TOP IS #, BOTTOM IS 2'S COMPLEMENT)

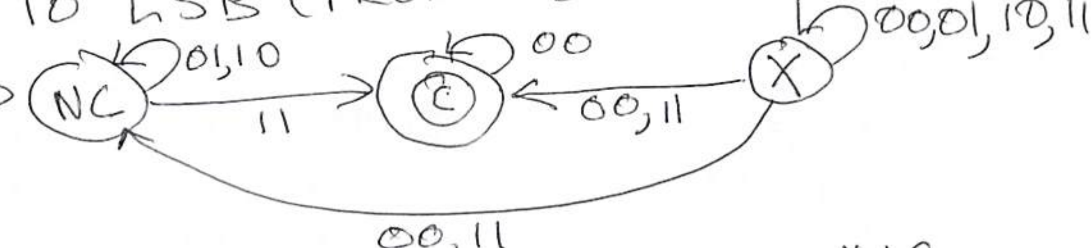
WE WILL DO LEAST SIGNIFICANT TO MOST SIGNIFICANT BIT (LSB TO MSB) FIRST

$Q = \{C, NC, X\}$ $q_0 = C$ $F = \{NC\}$

CARRY NO CARRY REJECT

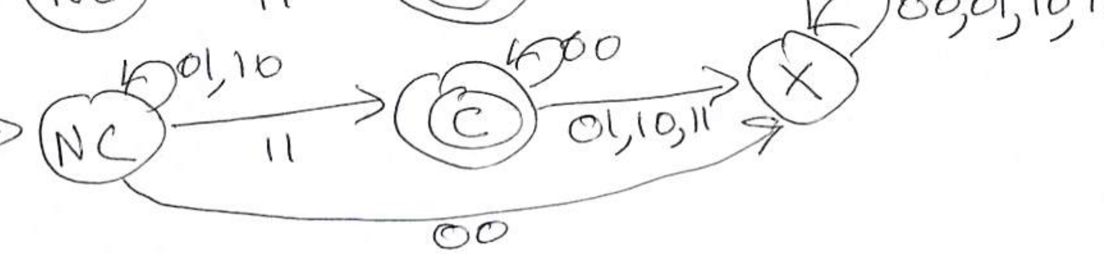
$Q_2 \rightarrow$


MSB TO LSB (FROM Q_2)

$Q_3 \rightarrow$


BUT X IS NOT REACHABLE FOM NC

$Q_3' \rightarrow$


OR $Q_3'' \rightarrow$


WEEK #2

NON-DETERMINISTIC AUTOMATON (NFA)

REALLY VIEW AS $\{q_0\}$

$$\mathcal{Q} = (Q, \Sigma, \delta, q_0, F)$$

$$\delta : Q \times \Sigma_\epsilon \to 2^Q = \mathcal{P}(Q)$$

$$\Sigma_\epsilon = \Sigma \cup \{\epsilon\} = \Sigma \cup \{\lambda\}$$

$$\lambda\text{-CLOSURE}(S) = \{t \mid t \in \delta^*(S, \lambda)\},\ S \in \mathcal{P}(Q)$$

HERE, WE USED EXTENDED δ THAT OPERATES ON SETS $\delta(S,a) = \bigcup_{q \in S} \delta(q,a)$ WHERE $a \in \Sigma_\epsilon$

THIS ACCOMMODATES STATE CHANGES WITHOUT READING ANY INPUT.

$$\delta^*(S,\lambda) = \lambda\text{-CLOSURE}(S)$$

$$\delta^*(S,ax) = \bigcup_{q \in S} \bigcup_{p \in \text{CLOSURE}(\delta(q,a))} \delta^*(p,x),\quad a \in \Sigma,\ x \in \Sigma^*,\ S \in \mathcal{P}(Q)$$

δ^+ AS BEFORE

$$L(\mathcal{Q}) = \{w \mid \delta^*(\{q_0\}, w) \cap F \neq \emptyset\}$$

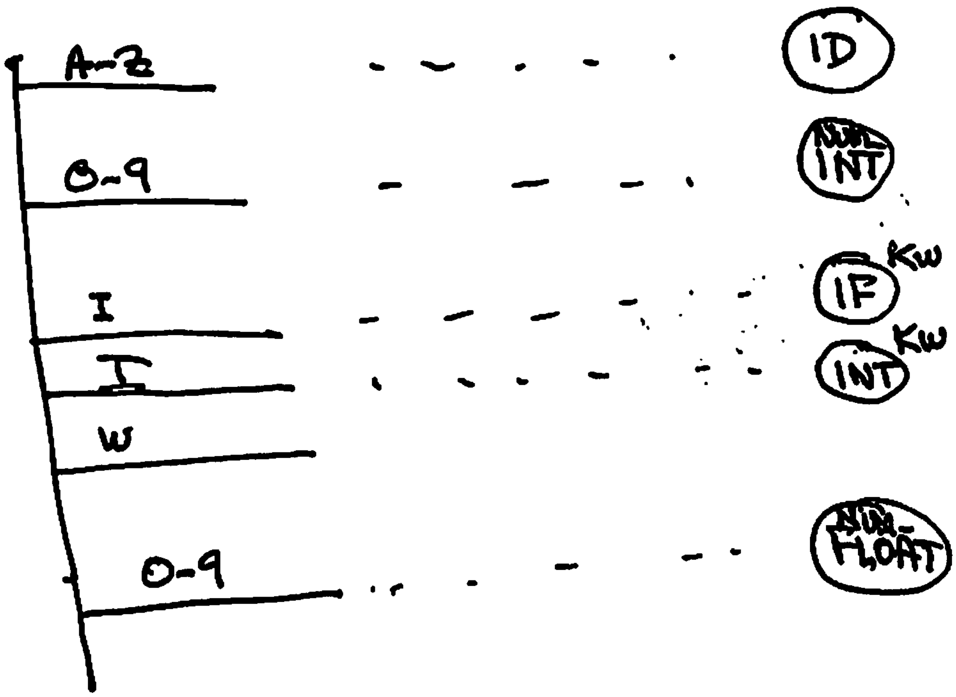
ACTUALLY, ~~BETTER~~ TO USE

$$L(\mathcal{Q}) = \{w \mid \delta^*(\lambda\text{-CLOSURE}(\{q_0\}), w) \cap F \neq \emptyset\}$$

OR EVEN CHANGE START TO A SET λ-CLOSURE$(\{q_0\})$

WEEK #2
CONSIDER LEXICAL ANALYSIS

ASSUME FIRST CHARACTER READ IN ADVANCE



WEEK #2

COMPLEMENT OF REGULAR LANGUAGE

1. REALLY BEGS FOR DFA vs NFA

$Q = (Q, \Sigma, \delta, q_0, F)$ A DFA

$\mathcal{L}(Q) = \{ w \mid \delta^*(q_0, w) \in F \}$

$\overline{\mathcal{L}(Q)} = \{ w \mid \delta^*(q_0, w) \notin F \}$

$= \{ w \mid \delta^*(q_0, w) \in Q - F \}$

Thus

$\overline{\mathcal{L}(Q)} = \mathcal{L}(Q^c)$ WHERE

$Q^c = (Q, \Sigma, \delta, q_0, Q - F)$

WHICH IS A DFA

2. $Q = (Q, \Sigma, \delta, q_0, F)$ AN NFA

$\mathcal{L}(Q) = \{ w \mid \delta^*(q_0, w) \cap F \neq \emptyset \}$

BUT $\mathcal{L}(Q') = \{ w \mid \delta^*(q_0, w) \cap (Q - F) \neq \emptyset \}$

IS NOT $\overline{\mathcal{L}(Q)}$

WEEK #2

WHEN SHOWING CLOSURE, WANT RIGHT MODEL

3. UNION

FOR NFA'S

$\mathcal{Q}_1 = (Q_1, \Sigma, \delta_1, q_{10}, F_1)$

$\mathcal{Q}_2 = (Q_2, \Sigma, \delta_2, q_{20}, F_2)$

CAN DO UNION OF $L(\mathcal{Q}_1)$ AND $L(\mathcal{Q}_2)$

BY

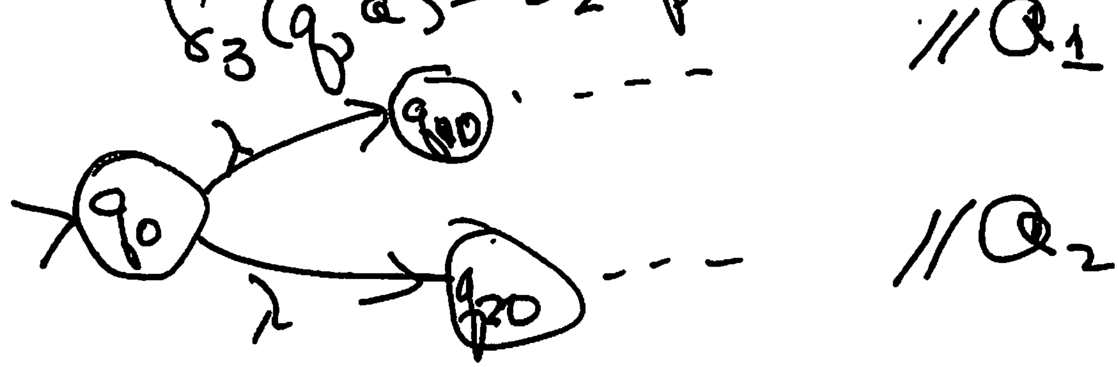
$\mathcal{Q}_3 = (\{q_0\} \cup Q_1 \cup Q_2, \delta_3, q_0, F_1 \cup F_2)$

WHERE

$\delta_3(q_0, \lambda) = \{q_{10}, q_{20}\}$

$\delta_3(q, a) = \delta_1(q, a)$ WHEN $q \in Q_1$

$\delta_3(q, a) = \delta_2(q, a)$ WHEN $q \in Q_2$



WEEK #2

4. UNION CAN BE DONE WITH DFA BY

$Q_1 = (Q_1, \Sigma, \delta_1, q_{10}, F_1)$; $Q_2 = (Q_2, \Sigma, \delta_2, q_{20}, F_2)$

$Q_3 = (Q_1 \times Q_2, \Sigma, \delta_3, \langle q_{10}, q_{20} \rangle, F_3)$

WHERE

$Q_1 \times Q_2 = \{ \langle p, q \rangle \mid p \in Q_1, q \in Q_2 \}$

$\delta_3(\langle p, q \rangle, a) = \langle \delta_1(p, a), \delta_2(q, a) \rangle$

SYNCHRONIZED PARALLEL
(MISD - MULTIPLE INSTRUCTION, SINGLE DATA)

$F_3 = F_1 \times Q_2 \cup Q_1 \times F_2$

ACCEPTS IF FIRST OR SECOND OR BOTH ACCEPT

5. CAN USE THIS CONSTRUCT FOR MANY SET OPERATIONS, SOME OF WHICH, LIKE COMPLEMENT, CANNOT BE DIRECTLY PROVEN WITH NFA MODEL

$\mathcal{L}(Q_1) \cap \mathcal{L}(Q_2)$ SET $F_3 = F_1 \times F_2$

$\mathcal{L}(Q_1) - \mathcal{L}(Q_2)$ SET $F_3 = F_1 \times (Q_2 - F_2)$

$\mathcal{L}(Q_1) \oplus \mathcal{L}(Q_2)$ SET $F_3 = F_1 \times (Q_2 - F_2) \cup (Q_1 - F_1) \times Q_2$

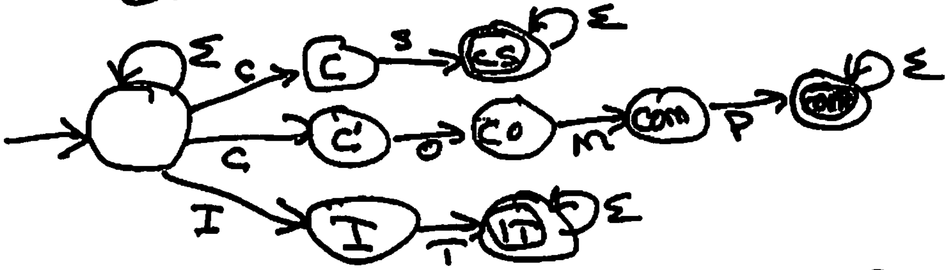
$= F_1 \times Q_1 \cup Q_1 \times F_2 - F_1 \times F_2$

6. OF COURSE, IF YOU HAVE UNION & COMPLEMENT, YOU CAN BUILD ALL OF ABOVE, BUT IT'S NOT AS INSTRUCTIVE OR INTUITIVE AS CONSTRCTION

WEEK #2

7. NFA'S

IF WANT ANY STRING CONTAINING "CS" OR "COMP" OR "IT"



GENEALLY EASIER TO WRITE NFA WHEN MULTIPLE PATHWAYS TO SUCCESS.

8. CONVERSION TO DFA

FIRST WANT TO ACCOMMODATE λ-TRANSITIONS

FOR $q \in Q$,

$$\lambda\text{-CLOSURE}(q) = \{t \mid \delta^*(q,\lambda) \supseteq \{t\}\}$$
$$= \{t \mid t \in \delta^*(q,\lambda)\}$$

FOR $S \subseteq Q$

$$\lambda\text{-CLOSURE}(S) = \{t \mid t \in \bigcup_{q \in S} \lambda\text{-CLOSURE}(q)\}$$

LET $A = (Q, \Sigma, \delta, q_0, F)$ BE AN NFA

DEFINE

$$A' = (\langle P(Q)\rangle, \Sigma, \delta', \langle \lambda\text{-CLOSURE}(q_0)\rangle, F')$$

WHERE

$$\delta'(\langle S\rangle, a) = \langle \lambda\text{-CLOSURE}(\delta(S,a))\rangle$$
$$= \langle \bigcup_{q \in S} \lambda\text{-CLOSURE}(\delta(q,a))\rangle, a \in \Sigma, S \in P(Q)$$
$$F' = \{\langle S\rangle \in \langle P(Q)\rangle \mid (S \cap F) \neq \emptyset\}$$

THIS LEADS TO $2^{|Q|}$ STATES, MOST OF WHICH ARE UNREACHABLE FROM START.

WEEK #2

9. COMPACT DFA FROM NFA

BUILD NEEDED SUBSETS FROM STATES ACCESSIBLE FROM START

PROCESS
COMPUTE λ-CLOSURE OF EACH $q \in Q$

START WITH λ-CLOSURE OF q_0 AND

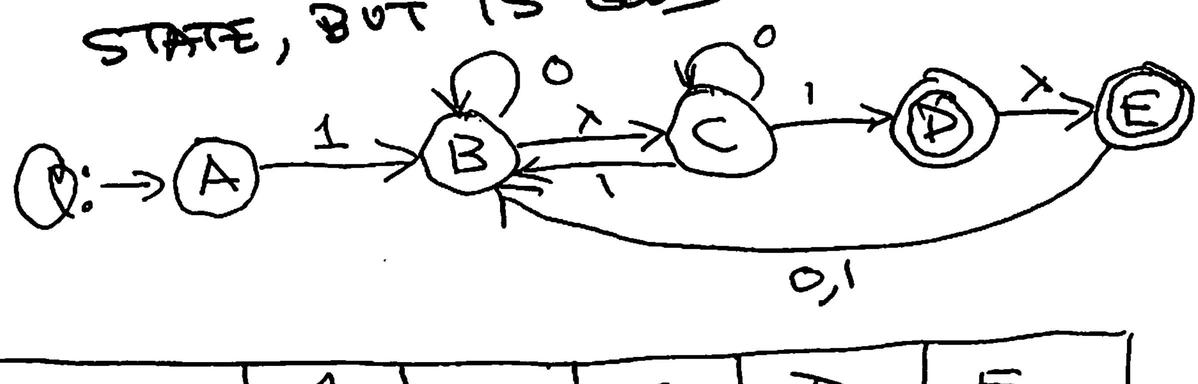
COMPUTE ALL SUCCESSOR STATES OF ONE STEP, ALWAYS DOING λ-CLOSURE OF THESE STATE SETS.

NEVER ADD AN UNNEEDED STATE

NOTE: $\langle \phi \rangle$ IS COMMON AND IS "DEAD" STATE

EVEN THIS DOES NOT GET MINIMAL STATE, BUT IS GOOD START

10.



STATE	A	B	C	D	E
λ-CLOSURE	{A}	{B,C}	{C}	{D,E}	{E}

WEEK#2