COT 4210 Fall 2016 **Midterm 2** **Name: ________________**
Total Points Available 60 **Your Raw Score _______** **Grade:____________**

2 **1.** Let **A = ({q1, … q10}, {0,1}, q1, {q3,q7})** be some DFA. Assume you have computed the sets, $R^k_{i,j}$, for $0 \le k \le 10, 1 \le i \le n, 1, 1 \le j \le n$. In terms of the $R^k_{i,j}$, what is the language recognized by **A**?

4 **2.** Write a Context Free Grammar for the language **L**, where
L = { w | w ∈ {a,b}* and w has twice as many a's as b's }.
Hint: You need to consider all possible relative orderings of **a**'s and **b**'s

5 **3.** Use Myhill-Nerode to show that the following language **L** is not Regular.
$L = \{ a^k\ b^{k^2} \mid k > 0 \}$.
Hint: Use the right-invariant equivalence relation R_L, where $x\ R_L\ y$ iff $\forall z[xz \in L \Leftrightarrow yz \in L]$

7 **4.** Consider the language

$L = \{ a^n b^n c^m \mid m > n \}$.
Use the Pumping Lemma for Context-Free Languages to show that **L** is not context-free.
Hint: It may be helpful to use different values of i for your various cases.

10 **5.** Present the **CKY** recognition matrix for the string **abbaab** assuming the Chomsky Normal Form grammar, **G = ({S,A,B,C,D,X }, {a,b}, R, S)**, specified by the rules **R**:

S → AB | BA
A → CX | a
B → XD | b
C → XA
D → BX
X → a | b

	a	**b**	**b**	**a**	**a**	**b**
1						
2						
3						
4						
5						
6						

Is **abbaab** in ***L*(G)?**___________

How do you know from above?________________________________

4 **6**. Give an explicit example of two Context Free Languages, L1 and L2, whose intersection is the non-Context Free Language { $\mathbf{a^n\ b^n\ c^n \mid n \geq 0}$ }. No grammars or proof is required.

L1 = ____

L2 = ____

Give an explicit example of a Regular Language, **R**, whose intersection with the Context Sensitive **L =** { $\mathbf{a^n\ b^n\ c^n \cup a^n\ b^n \mid n \geq 0}$ } is a Context Free, non-Regular Language. No grammars or proof is required, but you must describe the language produced by this intersection

R = ____

L ∩ R = ____

8 **7**. Prove that any class of languages, ***C***, closed under union, concatenation, intersection with regular languages, homomorphism and substitution (e.g., the Context-Free Languages) is closed under **huh** where **L** ∈ ***C***, **R** is Regular, **L** and **R** are over the alphabet **Σ**, and
L huh R = { y | y ∈ Σ^+ and ∃ **x,z ∈** R^+**,** such that **xyz ∈ L }.**
You may assume substitution **f(a) = {a, a'},** and homomorphisms **g(a) = a'** and **h(a) = a, h(a') =** λ.
Here **a∈Σ** and **a'** is a distinct new character associated with each **a∈Σ**.
You must be very explicit, describing what is produced by each transformation you apply and what kind of language results.

8 **8.** Consider the CFG **G = ({ S, T, C, D } , { a , b, c, d } , R , S)** where **R** is:

S → a T S | a T D
T → b T S | b C
C → c
D → d

a.) Present a pushdown automaton that accepts the language generated by this grammar. You may (and are encouraged) to use a transition diagram where transitions have arcs with labels of form **a,** $\alpha \rightarrow \beta$ where **a** $\in \Sigma \cup \{\lambda\}$, $\alpha, \beta \in \Gamma^*$. Note: You are allowed to use extended stack operations that push more than one symbol onto stack.

b.) What parsing technique are you using? (Circle one) **top-down** or **bottom-up**
How does your PDA accept? (Circle one) **final state** or **empty stack** or **final state and empty stack**
What is the **initial state**? ______________________
What is the **initial stack content**? ______________________
What are your **final states** (if any)? ______________________

c.) Now, using the notation of **ID**s (Instantaneous Descriptions, **[q, x, z]**), describe how your PDA accepts strings generated by **G**.

2 **9.** Consider the context-free language **L =** $\{ a^n b^m \mid n<m \}$. What language results when we take the **Max** of this language? What about the **Min**? To help you recall definitions, here they are.

Max(L) = { w | w $\in$ **L**, and if **wy** $\in$ **L**, then **y =** λ **}**
Max says that a string is kept only if that string is not a proper prefix of another string in L
Give your explicit answer for L = $\{ a^n b^m \mid n<m \}$ **below**
Max(L) =

Min(L) = { w | w $\in$ **L**, and if **xy = w** and **x** $\in$ **L**, then **y =** λ **}**
Min says that a string is kept only if no proper prefix of that string is in L
Give your explicit answer for L = $\{ a^n b^m \mid n<m \}$ **below**
Min(L) =

10. Consider the context-free grammar **G = ({ S, A, B }, { a,b }, R, S)**, where **R** is:

S → aAa | bBb
A → CA | AB
B → C | b
C → D | λ
D → abC

3 **a.)** Remove λ-rules from **G**, creating an equivalent grammar **G'**. Show all rules.
***Nullable* = { }**

2 **b.)** Remove all **unit** rules from **G'**, creating an equivalent grammar **G''**. Show all rules.
Chain(S) = { }; Chain(A) = { }; Chain(B) = { };
Chain (C) = { }; Chain(D) = { }

2 **c.)** Remove all useless symbols, creating an equivalent grammar **G'''**. Show all rules.
Unproductive = { }; Unreachable = { }

3 **d.)** Convert grammar **G'''** to its **Chomsky Normal Form** equivalent, $\mathbf{G^{iv}}$. Show all rules.