Assignment # 9.1a Key

1. **Use quantification of an algorithmic predicate to estimate the complexity (decidable, re, co-re, non-re) of each of the following, (a)-(d):**

ID = { f | $\forall$x f(x) = x}

$\forall$x $\exists$t [STP(f,x,t) & (VALUE(f,x,t) = x)]

Non-RE, Non-Co-RE

Assignment # 9.1b Key

b.) STUTTER = { f | for some x, f(x+1) = f(x) }

∃<x,t> [STP(f,x,t) & STP(f,x+1,t) &
(VALUE(f,x,t) = (VALUE(f,x+1,t))]

RE

Assignment # 9.1c Key

FIB1 = { f | for some x>1 f(x+2) = f(x+1) + f(x) }

$\exists$<x,t> [x>1 & STP(f,x,t) & STP(f,x+1,t) STP(f,x+2,t) & (VALUE(f,x+2,t) = VALUE(f,x+1,t) + VALUE(f,x,t))]

RE

Assignment # 9.1d Key

d) FIB2 = { f | there is some $x \geq 0$, such that for all $y \geq x$ [f(y+2) = f(y+1) + f(y)] }

$\exists x \forall y \ \exists t$ [y≥x & STP(f,y,t) & STP(f,y+1,t) STP(f,y+2,t) & (VALUE(f,y+2,t) = VALUE(f,y+1,t) + VALUE(f,y,t))]

Non-RE, Non-Co-RE

Assignment # 9.2 Key

2. Let sets A be recursive (decidable) and B be re non-recursive (undecidable).
   Consider $C = \{ z \mid z = \max(x,y), x \in A, y \in B \}$. For (a)-(c), either show sets A and B with the specified property or demonstrate that this property cannot hold.

a) Can C be recursive?

YES. Consider A = ℵ. B = Halt.
C = {x | x ≥ the smallest value in B}. As B is non-empty it has some smallest value, s, so C is well-defined and
$\chi_C(x) = x \geq s$

Assignment # 9.2b Key

b) Can C be non-recursive?

YES. Consider A = { 0 }. B = Halt. C = Halt. This is semi-decidable but non rec as Halt is equivalent to C.

Assignment # 9.2c Key

c) Can C be non-re?

No. Can enumerate C as follows.

First if A is empty then C is empty and so RE by definition.

If A is non-empty then A is enumerated by some algorithm f_A as recursive sets are RE.

As B is non-recursive RE, then it is non-empty and enumerated by some algorithm f_B.

Define f_C by $f_C(\langle x,y \rangle) = \max(f_A(x),f_B(y))$. f_C is clearly an algorithm as it is the composition of algorithms. The range of f_C is then $\{ z \mid \max(x,y)$, where $x \in A$ and $y \in B \} = C$ and so C must be RE.