Assignment # 8.1 Key

1. Use reduction from **HALT** to show that one cannot decide **STUTTER**, where **STUTTER = { f | for some x, f(x+1) = f(x) }**

Let f,x be an arbitrary pair of natural numbers. <f,x> is in Halt iff $\varphi_f(x)\downarrow$

Define g by $\varphi_g(y) = \varphi_f(x) - \varphi_f(x)$, for all y.

Clearly, $\varphi_g(y) = 0$, for all y, iff $\varphi_f(x)\downarrow$, and $\varphi_g(y)\uparrow$, for all y, otherwise.

Summarizing, <f,x> is in Halt implies g is in STUTTER and <f,x> is not in Halt implies g is not in STUTTER.

Halt $\leq_m$ STUTTER as we were to show.

Note: I have not overloaded the index of a function with the function in my proof, but I do not mind if you do such overloading.

Assignment # 8.2 Key

2. Show that STUTTER reduces to Halt. (1 plus 2 show they are equally hard)

Let f be an arbitrary natural number. f is in STUTTER iff for some x, $\varphi_f(x)\downarrow$, $\varphi_f(x+1)\downarrow$ and $\varphi_f(x) == \varphi_f(x+1)$

Define g by $\varphi_g(z) = \exists$ <x,t> [STP(f,x,t) & STP(f,x+1,t) & (VALUE(f,x,t) == (VALUE(f,x+1,t))], for all z.

Clearly, $\varphi_g(z) = 1$, for all z, iff there is some, x, such that $\varphi_f(x)\downarrow$ and $\varphi_f(x+1)\downarrow$ and $\varphi_f(x) == \varphi_f(x+1)$, and $\varphi_g(z)\uparrow$, for all z, otherwise.

Summarizing, f is in STUTTER iff <g,0> is in Halt and so

STUTTER $\leq_m$ Halt as we were to show.
Note that this also shows STUTTER $\leq_m$ Total but that's not surprising.

Assignment # 8.3 Key

3. Use Reduction from TOTAL to show that ID is not even re, where
   ID = { f | $\forall$x f(x) = x}

Let f be an arbitrary natural number. f is in Total iff $\forall$x $\varphi_f(x)\downarrow$

Define g by $\varphi_g(x) = \varphi_f(x) - \varphi_f(x) + x$, for all x.

Clearly, $\varphi_g(x) = x$ for all x, iff $\forall$x $\varphi_f(x)\downarrow$; otherwise $\varphi_g(x)\uparrow$ for some x.

Summarizing, f is in Total iff g is in ID and so

TOTAL $\leq_m$ ID as we were to show.

Assignment # 8.4 Key

4. Show ID reduces to Total. (3 plus 4 show they are equally hard)

Let f be an arbitrary natural number. f is in ID iff $\forall x\ \varphi_f(x) = x$.

Define g by $\varphi_g(x) = \mu y[\varphi_f(x) == x]$, for all x.

Clearly, $\varphi_g(x)\downarrow$, for all x, iff $\forall x\ \varphi_f(x) = x$; otherwise $\varphi_g(x)\uparrow$ for some x.

Summarizing, f is in ID iff g is in Total and so

ID $\leq_m$ TOTAL as we were to show.

Assignment # 8.5 Key

5. Use Rice's Theorem to show that STUTTER is undecidable

First, STUTTER is non-trivial as C0(x) = 0 is in STUTTER and S(x) = x+1 is not.

Second, STUTTER is an I/O property.

To see this, let f and g are two arbitrary indices such that
$\forall x\ [\varphi_f(x) = \varphi_g(x)]$

f $\in$ STUTTER iff $\exists$ x such that $\varphi_f(x) = \varphi_f(x+1)$
iff for some x_0, $\varphi_f(x_0) = \varphi_f(x_0+1)$
iff, since $\forall x\ [\varphi_f(x) == \varphi_g(x)]$, for some x_0, $\varphi_g(x_0) = \varphi_g(x_0+1)$
iff $\exists$ x such that $\varphi_g(x) = \varphi_g(x+1)$ iff g $\in$ STUTTER.

Thus, f $\in$ STUTTER iff g $\in$ STUTTER.

Assignment # 8.6 Key

6. Use Rice's Theorem to show that ID is undecidable

First, ID is non-trivial as I(x) = x is in ID and S(x) = x+1 is not.

Second, ID is an I/O property.

To see this, let f and g are two arbitrary indices such that $\forall x\ [\varphi_f(x) = \varphi_g(x)]$.

f $\in$ ID iff $\forall x\ \varphi_f(x)=x$
iff, since $\forall x\ [\varphi_f(x) = \varphi_g(x)]$, $\forall x\ \varphi_g(x) = x$ iff g $\in$ DOUBLES.

Thus, f $\in$ ID iff g $\in$ ID.