Assignment # 7.1 Key

1. Write a CFG to show the language is a CFL or use the Pumping Lemma for CFLs to prove that it is not for each of the following.

This Language is context free as can be seen from the following grammar.

a) $L = \{ x\, w\, w^R \mid w, x \in \{a,b\}^+ \}$

$S \rightarrow a\,S \mid b\,S \mid a\,X \mid b\,X$
$X \rightarrow a\,X\,a \mid b\,X\,b \mid a\,a \mid b\,b$

Assignment # 7.1b Key

1. Write a CFG to show the language is a CFL or use the Pumping Lemma for CFLs to prove that it is not for each of the following.

b) **$L = \{ a^n b^{sum(1..n)} \mid n>0 \}$**
Assume this language is a CFL
PL: Provides N>0
Me: $a^N b^{sum(1..N)}$
PL: $a^N b^{sum(1..N)} = uvwxy$, $|vwx| \leq N$, $|vx|>0$, and $\forall i\ uv^iwx^iy \in L$
ME: i=2.
Case 1) vwx contains some a's. Then uv^2wx^2y has at least N+1 a's and at most sum(1..N) + N-1 b's. But sum(1..N+1) is sum(1..N) + N+1 and so we do not have enough b's. Thus, $uv^2wx^2y \notin L$
Case 2) vwx contains only b's. Then uv^2wx^2y has exactly N a's and at least sum(1..N) + 1 b's. But then it has too many b's. Thus, $uv^2wx^2y \notin L$
These cases cover all possibilities, so L is not a CFL.

Assignment # 7.2 Key

2. Consider the context-free grammar **G = ({ S } , { a , b } , R , S),** where **R** is:

   $\mathbf{S} \;\rightarrow\; \mathbf{S\,a\,S\,b\,S} \;|\; \mathbf{S\,b\,S\,a\,S} \;|\; \boldsymbol{\lambda}$

   Provide the part of the proof that shows

   L = { w | w has as many **a's** as **b's }** $\subseteq$ **L (G)**

   You will need to provide an inductive proof.

   Show all strings in **L** are in **L(G)**. Note strings in **L** are all of even length.
   Base: **w** has **0 a**'s and **0 b**'s. Then **w =** $\boldsymbol{\lambda}$. Clearly **S** $\rightarrow$ $\boldsymbol{\lambda}$ and so S $\Rightarrow^*$ $\boldsymbol{\lambda}$ $\in$ **L(G).**
   IH: Assume all strings in **L** of length **2k or fewer**, **k≥0,** are in **L(G)**.
   IS: Any string **w**$\in$**L** of length **2K+2** must either start with an **a** or start with a **b**. if it starts with an **a**, then it must be of form **a x b y**, where **x** and **y** each have an equal number of **a**'s and **b**'s. Since **|x|** and **|y|** are each **≤ 2k**, by IH, **S** $\Rightarrow^*$ **x** and **S** $\Rightarrow^*$ **y**. But then
   S $\Rightarrow$ **S a S b S** $\Rightarrow$ **a S b S** $\Rightarrow^*$ **a x b y = w** $\in$ **L(G)**.
   This can easily be redone with **w = b x a y**, so **L** $\subseteq$ **L (G)**.