# Assignment # 5 (direct with M-N)

1. For each of the following, prove it is not regular by using the Pumping Lemma or Myhill-Nerode. You must do at least one of these using the Pumping Lemma and at least one using Myhill-Nerode.

a. $L = \{ xy \mid |x| = |y|; x \neq y; x, y \in \{0,1\}^* \}$

**I, personally, know of no way to attack this directly with the Pumping Lemma. One or more of you may and I will learn from that. However, I do know how to attack with Myhill-Nerode.**

**Let $R_L$ be the right invariant equivalence class defined by M-N for L.**
**Consider the pair of equivalent classes $[01^{2i}]$ and $[01^{2j}]$, $i \neq j$.**
**$01^{2i}01^{2i} \notin L$ because it is of the form xy, $|x|=|y|$, $x=y$, but $01^{2i}01^{2j} \in L$. To see this, consider two possibilities, $i>j$, $i<j$.**
**If $i>j$, then, for some k, $0<k\leq i$, $i - k = j$. Thus, $01^{2i}01^{2j} = 01^{2i-k} \mid 1^k01^{2j}$ and the midpoint is where I drew the |. The first half starts with a 0 and the second with a 1, and they are of the same length so it is in L.**
**If $i<j$, then, for some k, $0<k\leq j$, $i + k = j$. Thus, $01^{2i}01^{2j} = 01^{2i}01^{k-1} \mid 1^{2j-k+1}$ and the midpoint is where I drew the |. The first half starts with a 0 and the second with a 1, and they are of the same length so it is in L.**
**Thus, for each distinct pair, i,j, $i \neq j$, $[01^{2i}] \neq [01^{2j}]$ and hence $R_L$ has infinite index.**
**L is not Regular by Myhill-Nerode Theorem.**

# Assignment # 5.1a (indirect with PL)

1. For each of the following, prove it is not regular by using the Pumping Lemma or Myhill-Nerode. You must do at least one of these using the Pumping Lemma and at least one using Myhill-Nerode.

a. $L = \{ xy \mid |x| = |y|; x \neq y; x, y \in \{0,1\}^* \}$

**To attack this indirectly, we can note that the complement of L is made up of two non-overlapping languages L1 = { ww | w ∈ {0,1}* } and L2 = { w | |w is odd and w ∈ {0,1}* }. L2 is clearly Regular, but we will show L1 is not. We will need to be careful to not pump strings that are in L2.**

**Me: L1 ∪ L2 is regular**
**PL: Gives me N>0 associated with L1 ∪ L2**
**Me: Choose $w=01^{2N}01^{2N}$ which is in L1 but not L2**
**PL: States w=xyz, |xy|≤N, |y|>0, xyz ∈ L1 ∪ L2 for all i≥0**
**Me: Choose i = 2 (not i=0 as |y| might be odd). This says that $xy^2z = 01^{2N+2|y|}01^{2N} \in$ L1 ∪ L2 . Since this string is of even length, it cannot be in L2, so must be in L1, but it is not as it divides into two equal length strings $01^{2N}1^{|y|}$ | $1^{|y|}01^{2N}$ with the | as a dividing line and what is on the left starts with a 0 and on the right with a 1.**
**Thus, the complement of L is nor Regular and, since Regular are closed under complement, L cannot be Regular either.**

# Assignment # 5.1a (indirect with MH)

1. For each of the following, prove it is not regular by using the Pumping Lemma or Myhill-Nerode. You must do at least one of these using the Pumping Lemma and at least one using Myhill-Nerode.

**a.** **$L = \{ xy \mid |x| = |y|; x \neq y; x, y \in \{0,1\}^* \}$**

**To attack this indirectly, we can note that the complement of L is made up of two non-overlapping languages $L1 = \{ ww \mid w \in \{0,1\}^* \}$ and $L2 = \{ w \mid |w$ is odd and $w \in \{0,1\}^* \}$. L2 is clearly Regular, but we will show L1 is not. Let $R_{L1 \cup L2}$ be the right invariant equivalence class defined by M-N for $L1 \cup L2$.**
**Consider the pair of equivalent classes $[01^{2i}]$ and $[01^{2j}]$, i≠j.**
**$01^{2i}01^{2j} \notin L1 \cup L2$ because it is of even length and is not of the form xy, |x|=|y|, x=y, but $01^{2i}01^{2i} \in$ L and hence not to L1 and not to L2 as it is of even length.**
**Thus, for each distinct pair, i,j, i≠j, $[01^{2i}] \neq [01^{2j}]$ and hence $R_L$ has infinite index.**
**L is not Regular by Myhill-Nerode Theorem.**

**Note: using 2i and 2j is critical to be sure $01^{2i}01^{2j}$ has even length.**

# Assignment # 5.1b (M-N only)

1. For each of the following, prove it is not regular by using the Pumping Lemma or Myhill-Nerode. You must do at least one of these using the Pumping Lemma and at least one using Myhill-Nerode.

**b. L = { $a^i b^j c^k$ | i≥0, j≥0, k≥0, if i=1 then j>k }**

**I attack this with M-N. Let $R_L$ be the right invariant equivalence relation defined for L by M-H. Consider [$ab^i$] and [$ab^j$] i<j.**
**$ab^ic^j \in L$ but $ab^jc^j \notin L$ .**
**Thus, for any two distinct i,j, i≠j, [$ab^i$] ≠ [$ab^j$]**

**I do not know how to attack this directly with the Pumping Lemma – I could do so indirectly with the reversal of L, but I won't.**

# Assignment # 5.1c (PL—FAIL)

1. For each of the following, prove it is not regular by using the Pumping Lemma or Myhill-Nerode. You must do at least one of these using the Pumping Lemma and at least one using Myhill-Nerode.

c. $L = \{ x\, x\, w \mid x, w \in \{a,b\}^+ \}$

**Me: L is regular**
**PL: Gives me N>0 associated with L**
**Me: Choose $w=ab^Nab^{N+1}$ which is in L**
**PL: States w=xyz, |xy|≤N, |y|>0, $xyz \in L$ for all i≥0**
**Me: Choose i = 2 (not i=0 as the suffix part could eat up extra b's).**
**This says that $xy^2z = ab^{N+|y|}ab^{N+1} \in L$ or $xy^2z = (ab^{|y|-1})^2b^{N-|y|}ab^{N+1} \in L$**
**For the first case, since the replicated part must start with an 'a' and there are at least as many b's before the second 'a' than follow it, this cannot be in L.**
**Unfortunately, the second case can be of the form xxw, where $xx = (ab^{|y|-1})^2$ and $w = b^{N-|y|}ab^{N+1}$ .Thus, my great attempt failed. Note: It would also fail for i=0 on the second case as that would erase the 'a' and expose what could be 2 b's in a row for xx.**

# Assignment # 5.1c (M-N)

1. For each of the following, prove it is not regular by using the Pumping Lemma or Myhill-Nerode. You must do at least one of these using the Pumping Lemma and at least one using Myhill-Nerode.

c. $L = \{ x\,x\,w \mid x, w \in \{a,b\}^+ \}$

**I attack this with M-N. Let $R_L$ be the right invariant equivalence relation defined for L by M-H.**
**Consider $[ab^i]$ and $[ab^j]$ $i<j$.**
**$Ab^iab^{i+1} \in L$ but $ab^jab^{i+1} \notin L$, when $j>i$. The latter is true as the replicated parts must both start with 'a' and there are not enough b's to match the j b's ($j>i$) in the second case.**
**Thus, for any two distinct i,j, $i \neq j$, $[ab^i] \neq [ab^j]$**

**Thus, L is not Regular.**

# Assignment # 5.2

2. Write a regular (right linear) grammar that generates **L = { w | w ∈ {0,1}*** and **w** interpreted as a binary number has a remainder of 2 when divided by 5 } .

**<0> → 0 <0> | 1 <1>**

**<1> → 0 <2> | 1 <3>**

**<2> → 0 <4> | 1 <0> | λ**

**<3> → 0 <1> | 1 <2>**

**<4> → 0 <3> | 1 <4>**

# Assignment # 5.3

3. Present a Mealy Model finite state machine that reads an input **x** ∈ **{0, 1}*** and produces the binary number that represents the result of subtracting binary **10** from **x** (assumes all numbers are positive, including results). Assume that **x** is read starting with its least significant digit. Examples: **0010** → **0000; 1000** → **0110; 0001** → **1111** (wrong answer due to going negative, but nothing we can do about it.) Note: Can be attacked with 2's complement addition or direct subtraction.