Assignment 2 Key

Question 1)

Prove, if p and q are distinct prime numbers, then $\sqrt{(p/q)}$ is irrational.

Answer

Proof by contradiction :

Assume $\sqrt{\frac{p}{q}}$ is a rational number. Let $\frac{a}{b}$ be the reduced fraction (no common prime factors) that equals $\sqrt{\frac{p}{q}}$.

- $\sqrt{\frac{p}{q}} = \frac{a}{b} \Rightarrow \frac{p}{q} = \frac{a^2}{b^2} \Rightarrow a^2 q = pb^2$
- Because p and q are distinct prime numbers, a should have p as its factor => a = (kp)
- $(kp)^2 q = pb^2 \Rightarrow k^2 pq = b^2$ so b also needs to have p as its factor and it is a contradiction.

Question 2)

- Present a language L over Σ that has the following properties:
- $L \neq L^2$
- $L^2 = L^3$
- Note: $L^k = \{ x1x2...xk \mid x1,x2,...,xk \in L \}$.

Answer

- $\Sigma = \{x\}$
- $L = \Sigma^* - \{xx\} = \{\boldsymbol{\lambda}, x, xxx, xxxx, xxxxx, \dots\}$
- $L^2 = \boldsymbol{\lambda}(\Sigma^* - \{xx\}) \cup x(\{\boldsymbol{\lambda}, x, xxx, xxxx, xxxxx, \dots\}) \cup \dots$
  $= L \cup xx \cup \dots$
  $= \Sigma^*$
- $L^3 = \Sigma^* \Sigma^* = \Sigma^* = L^2$