Assignment 1 Key

Question 2)

Prove or disprove the following:

For non-empty sets A and B, (A-B)=A if and only if $A \cap B = \emptyset$.

A negative result must include sample sets A and B that contradict

the assertion. A supporting result must prove both directions as it's an iff property.

Answer

- Part 1) Prove if $A \cap B = \emptyset$ then $(A-B)=A$
- Proof by contradiction :

Suppose $A \cap B = \emptyset$ but $(A-B) \neq A$. If $(A-B) \neq A$ then, since $A \supseteq (A-B)$, we must assume that $\exists$ x such that $x \in A$ and $x \in B$, but then $x \in A \cap B$ and $A \cap B \neq \emptyset$, a contradiction.

Answer

- Part 2) Prove if (A-B)=A then A∩B = Ø
- Direct Proof:

Assume (A-B)=A then A ∩ ~B = A => A ∩ ~B ∩ B = A ∩ B => Ø = A ∩ B since ~B ∩ B = Ø and anything intersected with Ø is also Ø.
This shows that (A-B)=A implies A∩B = Ø as was desired.