

Assignment # 7.1 Sample

1. For the following languages, either provide a grammar to show it is a CFL or employ the Pumping Lemma to show it is not

a.) $L = \{ a^i b^j \mid j > 2 \cdot i \}$

b.) $L = \{ a^n b^{\text{Fib}(n)} \mid n > 0 \}$, where **Fib(i)** is the i^{th} Fibonacci number

Assignment # 7.2 Sample

2. Consider the context-free grammar $G = (\{ S \}, \{ a, b \}, S, P)$, where P is:

$$S \rightarrow S a S b S \mid S b S a S \mid S a S a S \mid a \mid \lambda$$

Provide the first part of the proof that

$$L(G) = L = \{ w \mid w \text{ has at least as many } a\text{'s as } b\text{'s} \}$$

That is, show that $L(G) \subseteq L$

To attack this problem we can first introduce the notation that, for a syntactic form α , $\alpha_a =$ the number of a 's in α , and $\alpha_b =$ the number of b 's in α . Using this, we show that if $S \Rightarrow^* \alpha$, then $\alpha_b \leq \alpha_a$ and hence that $L(G) \subseteq L$:

A straightforward approach is to show, inductively on the number of steps, i , in a derivation, that, if $S \Rightarrow^i \alpha$, then $\alpha_b \leq \alpha_a$.