Assignment # 1 Sample

1. Prove or disprove the following:
   For non-empty sets A and B, (AUB)=(A∩B) if and only if A=B

   Part 1) Prove if A = B, then (AUB)=(A∩B)

   Assume A=B then showing (AUB)=(A∩B) is equivalent to showing (AUA)=(A∩A). Now, any set unioned or intersected with itself is that set. Thus, (AUA) = A and (A∩A) = A and so (AUA)=(A∩A), proving that A = B implies (AUB)=(A∩B). Note: This is true even if both are empty.

   Part 2) Prove if (AUB)=(A∩B), then A = B

   Assume otherwise, then there is some case where (AUB)=(A∩B), but A≠B. This means one set must have an element that is missing from the other. As A's and B's roles are symmetric and each is non-empty, we can choose to say that there is some x in A that is not in B. As x is in A, it is in (AUB), but since it is not in B then it is not in (A∩B), and hence (AUB)≠(A∩B), but that contradicts our original assumption. Thus, (AUB)=(A∩B) implies A = B.

   Together Parts 1 and 2 show that, for non-empty A and B, (AUB)=(A∩B) if and only if A=B. Note: even here, the non-empty condition is superfluous as A≠B implies one has an element and we can just choose that one without worrying if the other is empty or not.