REGULAR EQUATIONS

Assume that $R$, $Q$ and $P$ are sets such that $P$ does not contain the string of length zero, and $R$ is defined by

$$R = Q + RP$$

We wish to show that

$$R = QP^*$$

We first show that $QP^*$ is contained in $R$. By definition, $R = Q + RP$.

To see if $QP^*$ is a solution, we insert it as the value of $R$ in $Q + RP$ and see if the equation balances

$$R = Q + QP^*P = Q(e+PP^*) = QP^*$$

Hence $QP^*$ is a solution, but not necessarily the only solution.

To prove uniqueness, we show that $R$ is contained in $QP^*$. By definition,

$$R = Q+RP = Q+(Q+RP)P = Q+QP+RP^2 = Q+QP+QP^2+RP^3 = \cdots = Q(e+P+P^2+ \cdots +P^i)+RP^{i+1}, \text{ for all } i\geq 0$$

Choose any $W$ in $R$, where the length of $W$ is equal to $k$. Then, from above,

$$R = Q(e+P+P^2+ \cdots +P^k)+RP^{k+1}$$

but, since $P$ does not contain the string of length zero, $W$ is not in $RP^{k+1}$. But then $W$ is in

$$Q(e+P+P^2+ \cdots +P^k)$$

and hence $W$ is in $QP^*$.

We use the above to solve simultaneous regular equations. For example, we can associate regular expressions with finite state automata as follows

$$A = B10^* + 0^*$$

$$B = B10^*1 + B0 + 0^*1$$

and therefore

$$B = 0^*1(10^*1 + 0)^*$$

Note: This technique fails if there are lambda transitions.