Non-Regular Languages

For each of the following, prove it is not regular by using the Pumping Lemma or Myhill-Nerode.

a. $\{ a^{k!} \mid k>0 \}$ This is set $\{a^1, a^2, a^6, a^{24}, a^{120}, \dots \}$

b. $\{ a^i b^j c^k \mid i \geq 0, j \geq 0, k \geq 0, j = i + k \}$

Pumping Lemma (k!)

1a. $\{ a^{k!} \mid k>0 \}$ using P.L.

1. Assume that L is regular
2. Let N be the positive integer given by the Pumping Lemma
3. Let *s* be a string $s = a^{(N+1)!} \in L$
4. Since *s* $\in$ L and $|s| \geq N$, s is split by PL into xyz, where $|xy| \leq N$ and $|y| > 0$ and for all $i \geq 0$, $xy^iz \in L$
5. We choose i = 2; by PL: $xy^2z = xyyz \in L$
6. Thus, $a^{(N+1)!+|y|}$ would be $\in$ L. This means that there is a factorial between (N+1)! and (N+1)!+N, but the smallest factorial after (N+1)! Is $(N+2)! = (N+2)\,(N+1)! = N(N+1)! + 2(N+1)! > (N+1)! + 2N > (N+1)!+N$
7. This is a contradiction, therefore L is not regular ■
8. Note: Using N is dangerous because N could be 1 and 2! is within N (1) of 1!

Pumping Lemma ($a^i b^j c^k$)

1b. { $a^i b^j c^k$ | $i \geq 0$, $j \geq 0$, $k \geq 0$, $j = i + k$ } using P.L.

1. Assume that L is regular
2. Let N be the positive integer given by the Pumping Lemma
3. Let *s* be the string $s = a^N b^N \in L$
4. Since $s \in L$ and $|s| \geq N$, s is split by PL into xyz, where $|xy| \leq N$ and $|y| > 0$ and for all $i \geq 0$, $xy^iz \in L$
5. We choose i = 0; by PL: $xz = xz \in L$
6. Thus, $a^{N-|y|}b^N$ would be $\in L$, but it's not since $N-|y| + 0 < N$.
   Note: The 0 is because there are 0 c's
7. This is a contradiction, therefore L is not regular ■

Myhill-Nerode (k!)

1a. $\{ a^{k!} \mid k>0 \}$ using M.N.

We consider the collection of right invariant equivalence classes $[a^{j!-j}]$, $j \geq 0$.

It's clear that $a^{j!-j}a^j$ is in the language,

but $a^{k!-k}a^j$ is not when $j < k$

This shows that there is a separate equivalence class $[a^{j!-j}]$ induced by R_L, for each $j \geq 0$.

Thus, the index of R_L is infinite and Myhill-Nerode states that L cannot be Regular. ■

Myhill-Nerode ($a^ib^jc^k$)

1b. { $a^ib^jc^k$ | $i\geq0$, $j\geq0$, $k\geq0$, $j = i + k$ } using M.N.
We consider the collection of right invariant equivalence classes $[a^j]$, $j \geq 0$.
It's clear that a^jb^j is in the language,
but a^kb^j is not when $j \neq k$
This shows that there is a separate equivalence class $[a^j]$ induced by R_L, for each $j \geq 0$.
Thus, the index of R_L is infinite and Myhill-Nerode states that L cannot be Regular. ■