Languages

Grammar

CFL
DSEL
REG

CFG
DCFG
LR(k)
LL(k)

K. Knuth
LR(1) = DCFL
1965
LL(k+1) ≠ LL(k)

R \rightarrow LL(k) = DCFL
1969

Early work: Precedence grammars 1963

LR
Left-to-right
Reverse rightmost deriv.
(Left is deferred to bottom of tree)

LL
Left-to-right
Leftmost deriv.
CNF

A → a
A → AB

IF \( \lambda \in L \) THEN

S → \( \lambda \)
BUT S NEVER ON RHS OF ANY RULE

CONTROLLED GROWTH IN TREE
(BASIS OF PUMPING LEMMA)

DIVIDE & CONQUER
(BASIS OF DYNAMIC PROGR. PARSER)
Conversion to CNF

1. **Remove nulling rules** (A→λ)
2. **Remove unit (chain) rules** (A→B)
3. **Remove non-productive non-terminals**
   \[ \text{Productive if } A^* \Rightarrow w \quad w \in \Sigma^* \]
4. **Remove unreachable symbols**
   \[ S^* \Rightarrow \epsilon A \] then A is **reachable**

**Note:** (1) Can create instances of (2)
1. Assignment #7 (c)

2. Pumping Lemma - Conceptually for \( n+1 \) Internal Nodes

   (a) Proof via derivations

   \[ S \Rightarrow^* \text{unty} \Rightarrow^* \text{untxy} \Rightarrow^* \text{unwxy} \]

   \[ A \Rightarrow T \Rightarrow^* \text{nTx} + T \Rightarrow^* W \]

   \[ \text{AND} T \Rightarrow \text{nTxi} \Rightarrow^* 0 \]

   (b) Tree for visualization

3. \( a^n b^n c^n \) not CFL

4. Note \( a^n b^n c^n = a^n b^n c^* \cup a^* b^n c^n \)

   so CFL not closed under \( \cup \)

   Also since if CFL were closed under complement, we can get \( \cup \)

   using complement \& \( \cup \), so

   CFL not closed under complement
5. \text{MAX}

L1 = \{a^ib^jck \mid k \leq i \text{ or } k \leq j \}

\text{max}(L1) = a^b c \text{max}(i,j)

Show above is not a CFL

Assume it is

PL: N > 0
ME: \alpha \in \Sigma \cup \Gamma \subseteq L
\Rightarrow |\alpha| \leq N, |\alpha| > 0

ME:

CASE 1: \text{NWx contains no c's} AS |\text{NWx}| > 0, \text{NWx contains either some a's, some b's or both a's+b's}

LET \text{L} = 2 \text{ then we either have more than N a's, more than N b's or more than N of each and so \#c's is not max of \#a's+\#b's} \text{ UN}\text{Wxwy} \notin L

CASE 2: \text{NWx contains some c's, it might contain b's but cannot contain a's AS |\text{NWx}| \leq N.}

LET \text{L} = 0 \text{ then we have fewer c's than a's and we have at most N b's}

SO \text{UWY = UNWxwy} \notin L

This covers all cases

Thus CFL not closed under MAX
6. \[ \operatorname{min} \]
   \[ L_2 = \{ aibic \mid k \geq i \text{ or } k \geq j \} \]
   \[ \operatorname{min}(L_2) = a^i b^j c^{\min(i,j)} \]
   Show above is not a CFL

   Assume it is
   PL: \[ N \geq 0 \]
   ME: \[ a^n b^m c^n \in L \]
   PL: \[ a^n b^m c^n = u_0 w_0 x_0 y_0 z_0, \quad \|x_0\| \leq N, \quad \|x_0\| \geq 0 \]
   \& \quad \text{for all } u_0 w_0 x_0 y_0 z_0 \in L

   ME:
   \text{Case 1: } nx \text{ contains no } c's

   Case 2: \( nx \) contains some \( c's \)

   Thus CFL not closed under \( \operatorname{min} \)
7. \( L_3 = \{ w w \mid w \in \{a, b\}^+ \} \)

Show above is not a CFL

Assume it is

\( P.L. \): \( n > 0 \)

\( M.E. \): \( a^n b^n a^n b^n \)

\( P.L. \): \( a^n b^n a^n b^n = \text{a valid string} \) \( \Rightarrow n > |w| \geq 0 \) \( \& \) \( n \geq 0 \) \( \text{such that } x_i y \leq L \)

\( M.E. ! \)

**Case 1:** \( n x \) contains some \( a \)'s
As \( |n w x| \leq n \), \( n w x \) must only have \( a \)'s from 1st or 2nd subsequence of \( a \)'s
But not both
Let \( l = 0 \) then one of the blocks of \( a \)'s separated by \( b \)'s has fewer \( a \)'s than the other and so \( w y \neq u^n \) \( \text{for } x \neq y \) \( \& L \)

**Case 2:** Same as above but focused on \( b \)'s

Thus \( L_3 \) not a CFL

Note from earlier \( L_3 \) is a CFL.

So CFLs are again shown to not be closed under complement.
8. SOLVABLE CFL PROBLEMS

WGL?
L = ∅ or L ≠ ∅
L finite or L infinite

9. MORE CLOSURE

INTERSECTION WITH REGULAR
SKETCH PROOF ONLY

SUBSTITUTION
CHANGE EVERY a ∈ Σ IN RHS
OF RULES TO S_0 WHERE
S_0 IS START SYMBOL OF G_0
AND L(G_0) = L_0 WHERE
f(a) = L_0

10. GO OVER PRIOR CLOSURES BY

OP(L) = Υ (f(L) ∩ REGEX)
WHERE f(a) = \{ a, a' \} if f(a) = a'
\& \ f(a) = a, a \in Σ

PREFIX, SUFFIX, SUBSTRING, ETC.
ALSO QUOTIENT WITH REGULAR