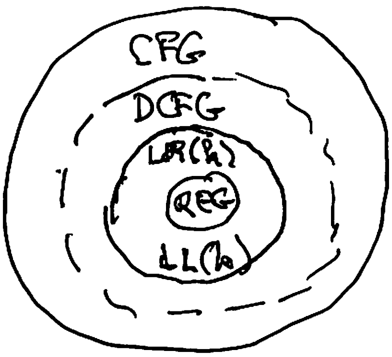
LANGUAGES GRAMMARS



KNUTH

LR(1) = DCFL

1965

LL(k+1) $\supsetneq$ LL(k)

k→∞ LL(k) = DCFL

1969

EARLY WORK: PRECEDENCE GRAMMARS 1963

LR LEFT-TO-RIGHT
REVERSE RIGHTMOST DERIV.
(LEFT IS DEFERRED TO BOTTOM
OF TREE)

LL LEFT-TO-RIGHT
LEFTMOST DERIV.

CNF

$A \to a$

$A \to AB$

IF $\lambda \in L$ THEN

$S \to \lambda$

BUT S NEVER ON RHS OF ANY RULE

CONTROLLED GROWTH IN TREE
(BASIS OF PUMPING LEMMA)

DIVIDE & CONQUER
(BASIS OF DYNAMIC PROGR. PARSER)

CONVERSION TO CNF

1. REMOVE NULLING RULES ($A \to \lambda$)

2. REMOVE UNIT (CHAIN) RULES ($A \to B$)

3. REMOVE NON-PRODUCTIVE NON-TERMINALS
   PRODUCTIVE IF $A \stackrel{*}{\Rightarrow} w \quad w \in \Sigma^*$

4. REMOVE UNREACHABLE SYMBOLS
   $S \stackrel{*}{\Rightarrow} \alpha A \beta$ THEN A IS REACHABLE

NOTE: (1) CAN CREATE INSTANCES OF (2)

1. ASSIGNMENT #7 (a)

2. PUMPING LEMMA - CONCEPTULLY FOR $|V|+1$ INTERNAL NODES

   (a) PROOF VIA DERIVATIONS

   $S \overset{*}{\Rightarrow} uTy \overset{*}{\Rightarrow} uvTxy \overset{*}{\Rightarrow} uvwxy$

   AND $T \overset{*}{\Rightarrow} vTx$ & $T \overset{*}{\Rightarrow} w$

   AND $T \overset{*}{\Rightarrow} v^i T x^i \quad i \geq 0$

   (b) TREE FOR VISUALIZATION

3. $a^n b^n c^n$ NOT CFL

4. NOTE $a^n b^n c^n = a^n b^n c^* \cap a^* b^n c^n$

   SO CFL NOT CLOSED UNDER $\cap$

   ALSO SINCE IF CFL WERE CLOSED UNDER COMPLEMENT, WE CAN GET $\cap$ USING COMPLEMENT & $\cup$, SO CFL NOT CLOSED UNDER COMPLEMENT

5. MAX

$L1 = \{a^i b^j c^k \mid k \le i \text{ OR } k \le j\}$

$max(L1) = a^i b^j c^{max(i,j)}$

SHOW ABOVE IS <u>NOT</u> A CFL

ASSUME IT IS

P.L.: $N > 0$

ME: $a^N b^N c^N \in L$

ME: $a^N b^N c^N = uvwxy$, $|vwx| \le N$, $|vx| > 0$

PL: & $\forall i \ge 0$ $uv^i w x^i y \in L$

ME:

CASE 1: vwx CONTAINS NO c's

AS $|vx| > 0$, vx CONTAINS EITHER SOME a's, SOME b's ~~OR~~ BOTH a's + b's

LET $i = 2$ THEN WE ~~EITHER HAVE~~ MORE THAN N a's, MORE THAN N b's OR MORE THAN N OF EACH AND SO #c's IS NOT MAX OF #a's + #b's & $uv^2wx^2y \notin L$

CASE 2: vx CONTAINS SOME c's, IT MIGHT CONTAIN b's BUT CANNOT CONTAIN a's

AS $|vwx| \le N$.

LET $i = 0$ THEN WE HAVE FEWER c's THAN a's AND WE HAVE AT MOST N b's

SO $uwy = uv^0wx^0y \notin L$

THIS COVERS ALL CASES

THUS CFL NOT CLOSED UNDER MAX

6. MIN

$L2 = \{a^i b^j c^k \mid k \geq i \text{ OR } k \geq j\}$

$MIN(L2) = a^i b^j c^{min(i,j)}$

SHOW ABOVE IS NOT A CFL

ASSUME IT IS

P.L.: $N > 0$

M.E: $a^N b^N c^N \in L$

P.L.: $a^N b^N c^N = uvwxy$, $|vwx| \leq N$, $|vx| > 0$

& $\forall i \geq 0$ $uv^i w x^i y \in L$

ME:

CASE 1: vwx CONTAINS NO c's

CASE 2: vx CONTAINS SOME C's

THUS CFL NOT CLOSED UNDER MIN

7. $L3 = \{ww \mid w \in \{a,b\}^+\}$

SHOW ABOVE IS NOT A CFL

ASSUME IT IS

P.L.: $N > 0$

ME: $a^N b^N a^N b^N$

P.L: $a^N b^N a^N b^N = uvwxy$, $|vwy| \le N$, $|vx| > 0$

& $\forall i \geq 0$ $uv^iwx^iy \in L$

ME:

CASE 1: vx CONTAINS SOME a'S

AS $|vwx| \le N$, vwx MUST ONLY HAVE a'S FROM 1ST OR 2ND SUBSEQUENCE OF a'S BUT NOT BOTH

LET $i = 0$ THEN ONE OF THE BLOCKS OF a'S SEPARATED BY b'S HAS FEWER a'S THAN THE OTHER AND SO $uwy = uv^0wx^0y \notin L$

CASE 2: SAME AS ABOVE BUT FOCUSED ON b'S

THUS L3 NOT A CFL

NOTE FROM EARLIER $\overline{L3}$ IS A CFL

SO CFLs ARE AGAIN SHOWN TO NOT BE CLOSED UNDER COMPLEMENT

8. SOLVABLE CFL PROBLEMS

$w \in L$?

$L = \emptyset$ OR $L \neq \emptyset$

L FINITE OR L INFINITE

9. MORE CLOSURE

INTERSECTION WITH REGULAR

SKETCH PROOF ONLY

SUBSTITUTION

CHANGE EVERY $a \in \Sigma$ IN RHS OF RULES TO S_a WHERE S_a IS START SYMOL OF G_a AND $\mathcal{L}(G_a) = L_a$ WHERE

$f(a) = L_a$

10. GO OVER PRIOR CLOSURES BY

$OP(L) = h(f(L) \cap REGEXPR)$

WHERE $\left.\begin{array}{l} f(a) = \{a, a'\} \quad g(a) = a' \\ \& \; h(a) = a, h(a') = \lambda \end{array}\right\} a \in \Sigma$

PREFIX, SUFFIX, SUBSTRING, ETC.

ALSO QUOTIENT WITH REGULAR