

Total Points Available ??

Your Raw Score _____

Grade: _____

1. Present the transition diagram for a **DFA** that accepts the set of binary strings that represent the magnitude of numbers that have a remainder of 1, when divided by 3. Numbers are read most to least significant digit, so 01 (1), 111 (7) and 10011 (19) are accepted, but 0000 (0), 110 (6) and 010001 (17) are not. Note: Leading zeroes are allowed.

2. Consider the following assertion:

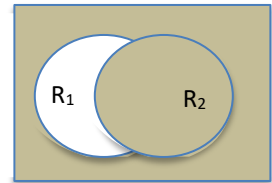
Let R_1 and R_2 be regular languages that are recognized by A_1 and A_2 , respectively, where

$A_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $A_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ are **DFA**s.

Show that $L = \sim(R_1 - R_2) = \{ w \mid w \text{ is in the complement of } (R_1 - R_2) \} =$

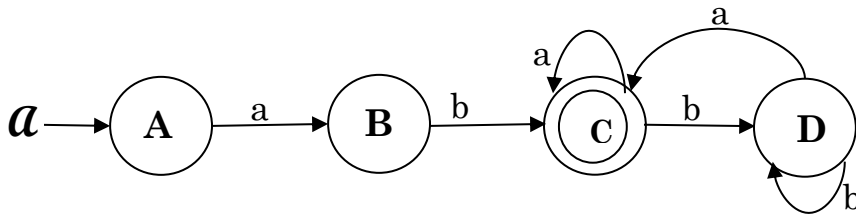
$$\sim(R_1 \cap \sim R_2) = \sim R_1 \cup R_2$$

is also regular, where \sim means set complement and $-$ means set difference.



Present a **DFA** construction $A_3 = (Q_3, \Sigma, \delta_3, q_3, F_3)$, where $L(A_3) = \sim(R_1 - R_2)$. You must clearly define Q_3 , δ_3 , q_3 , and F_3 . However, I do not require you to prove or even justify your choice of final set, just to get it right.

3. Let L be defined as the language accepted by the finite state automaton \mathcal{A} :



- a.) Present the regular equations associated with each of \mathcal{A} 's states, solving for the regular expression associated with the language recognized by \mathcal{A} .

- b.) Assuming that we designate **A** as state **1**, **B** as state **2**, **C** as state **3** and **D** as state **4**. Kleene's Theorem allows us to associate regular expressions $R_{i,j}^k$ with \mathcal{A} , where $i \in \{1..4\}$, $j \in \{1..4\}$, and $k \in \{0..4\}$.

The following are values of

$$R_{1,3}^3 = aba^* , R_{2,3}^3 = ba^* , R_{3,3}^3 = a^* , R_{3,4}^3 = b$$

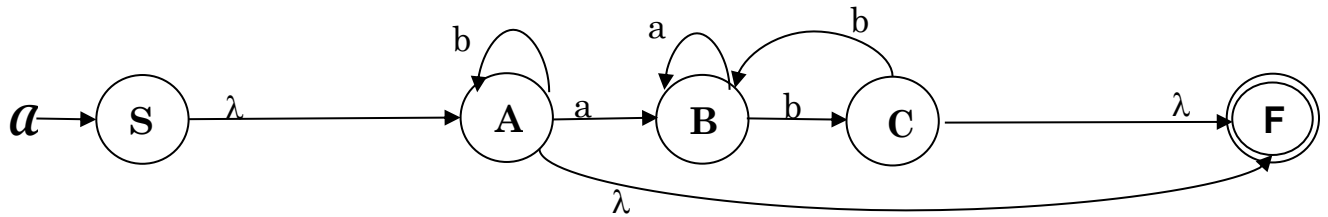
What are the values of the following?

$$R_{4,3}^3 = \quad , R_{4,4}^3 =$$

How is $R_{3,3}^4$ calculated from the set of $R_{i,j}^3$'s above? Give this abstractly in terms of the $R_{i,j}^3$'s

What expression does $R_{3,3}^4$ evaluate to, given that you have all the component values?

4. Let L be defined as the language accepted by the NFA \mathcal{A} :



Using the technique of replacing transition letters by regular expressions and then ripping states from a GNFA to create new expressions, develop the regular expression associated with the automaton \mathcal{A} that generates L . I have included the states of GNFA's associated with removing states A , B and then C , in that order. You must use this approach of collapsing one state at a time, showing the resulting transitions with non-empty regular expressions.

5. Consider the regular grammar $G = (\{S, A, B\}, \{0, 1\}, S, P)$ where P is the set of rules:

$$\begin{array}{l}
 S \rightarrow 1B \mid 1A \\
 A \rightarrow 0B \mid 1S \mid 1 \\
 B \rightarrow 0S \mid \lambda
 \end{array}$$

Present an NFA \mathcal{A} that accepts the language generated by G :

6. Analyze the following language, L , proving it is **non-regular** by showing that there are an **infinite** number of equivalence classes formed by the relation R_L defined by:

$$x R_L y \text{ if and only if } [\forall z \in \{a,b\}^*, xz \in L \text{ exactly when } yz \in L].$$

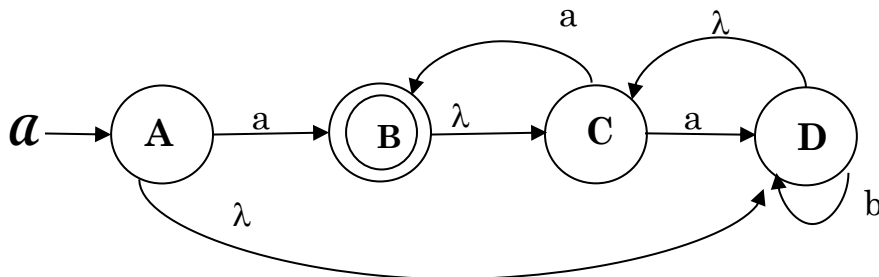
where

$$L = \{ a^i b^j \mid i < j \},$$

You don't have to present all equivalence classes, but you must demonstrate a pattern that gives rise to an infinite number of classes, along with evidence that these classes are distinct from each other.

7. Let L be defined as the language accepted by the finite state automaton \mathcal{A} :

- a.) Fill in the following table, showing the λ -closures for each of \mathcal{A} 's states.



State	A	B	C	D
λ -closure				

- b.) Convert \mathcal{A} to an equivalent deterministic finite state automaton. Use states like AC to denote the subset of states $\{A,C\}$. Be careful -- λ -closures are important.

8. **OddLetters(L)** = { $x_1 x_3 \dots x_{2n+1}$ | where $x_1 x_2 x_3 \dots x_{2n} x_{2n+1} \in L$ or $x_1 x_2 x_3 \dots x_{2n} x_{2n+1} x_{2n+2} \in L$ },
 where each $x_i \in \Sigma$

Show that any class of languages (in particular Regular and Context Free Languages) that is closed under substitution, concatenation and intersection with Regular Languages is also closed under **OddLetters**. A constructive solution is all I ask; no proof required. I'll help. Consider using substitutions $f(a) = \{a, a'\}$; $g(a) = a'$; $h(a) = a$, $h(a') = \lambda$, where $a \in \Sigma$ and a' is a new symbol uniquely associated with the symbol a .

9. Given a DFA denoted by the transition table shown below, and assuming that **1** is the start state and **3** and **6** are final states, fill in the equivalent states matrix I have provided. Use this to create an equivalent, minimal state DFA.

	a	b	c
>1	4	6	5
2	1	6	5
<u>3</u>	2	3	3
4	1	3	5
5	1	3	6
<u>6</u>	1	3	6

2					
<u>3</u>					
4					
5					
<u>6</u>					
	1	2	<u>3</u>	4	5

10. Write a **Context Free Grammar** for the language

$$L = \{ a^k b^m c^n \mid k = n + m, \text{ or } m = k + n, \text{ or } n = k + m, k > 0, m > 0, n > 0 \}.$$

11. Consider the language

$$L = \{ a^n b^{n!} \mid n > 0 \}.$$

Use the Pumping Lemma for Context-Free Languages to show that **L** is not context-free.

12. Present the **CKY** recognition matrix for the string **bbabb** assuming the Chomsky Normal Form grammar, $G = (\{S,A,B,C,D\}, \{a,b\}, R, S)$, specified by the rules **R**:

$$S \rightarrow AB \mid BA \mid BD$$

$$A \rightarrow CS \mid CD \mid a$$

$$B \rightarrow DS \mid b$$

$$C \rightarrow a$$

$$D \rightarrow b$$

		b	b	a	b	b
1						
2						
3						
4						
5						

13. Consider the context-free grammar $G = (\{S, A, B\}, \{a, b\}, R, S)$, where R is:

$$S \rightarrow SAB \mid BA$$

$$A \rightarrow AB \mid a$$

$$B \rightarrow bS \mid b \mid \lambda$$

a.) Remove all λ -rules from G , creating an equivalent grammar G' . Show all rules.

$$\text{Nullable} = \{ \quad \}$$

$$G' =$$

b.) Remove all **unit** rules from G' , creating an equivalent grammar G'' . Show all rules.

$$\text{Unit}(S) = \text{Chain}(S) = \{ \quad \}; \text{Unit}(A) = \{ \quad \}; \text{Unit}(B) = \{ \quad \}$$

$$G'' =$$

c.) Convert grammar G'' to its **Chomsky Normal Form** equivalent, G''' . Show all rules.

$$G''' =$$

In exam I may have some Unproductive non-terminals and some Unreachable ones.

14. Assume A and B are arbitrary Context Free languages. Indicate, for each of the following operations, whether the language L is guaranteed to be Context Free (Note: Regular languages are Context Free). No proofs or examples are required.

Operation	Is L guaranteed to be a CFL? (Y or N)
$L \subset A$ (Subset)	
$L = A \cap B$ (Intersection)	
$L = A \bullet B$ (Concatenation)	
$L = A \oplus B$ ($\{x \mid x \text{ is in either } A \text{ or } B, \text{ but not both}\}$)	
$L = \text{Max}(A)$ ($\{x \mid x \in A \text{ but no } xy \in A, y > 0\}$)	