

Sample Assignment # 5.1

1. For each of the following, prove it is not regular by using the Pumping Lemma or Myhill-Nerode. You must do at least one of these using the Pumping Lemma and at least one using Myhill-Nerode.
 - a. $\{ a^{2^{k+1}} \mid k \geq 0 \}$ (note: 2^{k+1} , so get $\{a^2, a^3, a^5, a^9, a^{17}, \dots\}$)
 - b. $\{ a^i b^j c^k \mid i \geq 0, j \geq 0, k \geq 0, \text{ if } i=0 \text{ then } j=2k \}$
 - c. $\{ xyz \mid x, y, z \in \{a, b\}^* \text{ and } y = xz \}$

Sample Assign # 5.1 Answer

1a. $\{a^{2^k+1} \mid k \geq 0\}$ using P.L.

1. Assume that L is regular
2. Let N be the positive integer given by the Pumping Lemma
3. Let s be a string $s = a^{2^N+1} \in L$
4. Since $s \in L$ and $|s| \geq N$, s is split by PL into xyz , where $|xy| \leq N$ and $|y| > 0$ and for all $i \geq 0$, $xy^iz \in L$
5. We choose $i = 2$; by PL: $xy^2z = xyyz \in L$
6. Thus, $a^{2^N+1+|y|}$ would be in L . This means that there is number in L between a^{2^N+1} and a^{2^N+1+N} , but next number after 2^N+1 is $2^{(N+1)}+1$. The distance is 2^N between these number and 2^N is greater than N for all values of N , meaning $a^{2^N+1+|y|}$ cannot be in L .
This is a contradiction, therefore L is not regular ■

1b. $\{a^i b^j c^k \mid i \geq 0, j \geq 0, k \geq 0, \text{ if } i=0 \text{ then } j=2k\}$ using P.L.

1. Assume that L is regular
2. Let N be the positive integer given by the Pumping Lemma
3. Let s be the string $s = b^{2N} c^N \in L$
4. Since $s \in L$ and $|s| \geq N$, s is split by PL into xyz , where $|xy| \leq N$ and $|y| > 0$ and for all $i \geq 0$, $xy^iz \in L$
5. We choose $i = 0$; by PL: $xy^0z = xz \in L$
6. Thus, $b^{2N-|y|} c^N$ would be in L , but it's not since $2N-|y| < 2N$
7. This is a contradiction, therefore L is not regular ■

Sample Assign # 5.1 Answer

1c. $\{ xyz \mid x,y,z \in \{a, b\}^* \text{ and } y = xz \}$ using P.L.

1. Assume that L is regular
2. Let N be the positive integer given by the Pumping Lemma
3. Let s be the string $s = a^N b a^N b \in L$
4. Since $s \in L$ and $|s| \geq N$, s is split by PL into xyz , where $|xy| \leq N$ and $|y| > 0$ and for all $i \geq 0$, $xy^i z \in L$
5. We choose $i = 0$; by PL: $xy^0 z = xz \in L$
6. Thus, $a^{N-|y|} b a^N b$ would be in L .
One b has to be part of x and the other of y , or of y and z . If one b in x then, since $N-|y| \neq N$, this is not of the proper form. If the b 's are in y and z , then we encounter the same issue.
7. This is a contradiction, therefore L is not regular ■

Sample Assign # 5.1 Answer

1a. $\{a^{2^k+1} \mid k \geq 0\}$ using M.N.

We consider the collection of right invariant equivalence classes $[a^{2^i+1}]$, $i \geq 0$.

It's clear that $a^{2^i+1} a^{2^i} = a^{2^{(i+1)+1}}$ is in the language, but $a^{2^j+1} a^{2^i} = a^{2^j+2^i+1}$ is not as 2^j+2^i is not a power of two when $i \neq j$. To see this, assume wlog that $j > i$, then the next power of two after 2^j is $2^{(j+1)} = 2^j + 2^j > 2^j + 2^i$.

This shows that there is a separate equivalence class $[a^{2^j+1}]$ induced by R_L , for each $j > 2$.

Thus, the index of R_L is infinite and Myhill-Nerode states that L cannot be Regular. ■

1b. $\{a^i b^j c^k \mid i \geq 0, j \geq 0, k \geq 0, \text{ if } i=0 \text{ then } j=2k\}$ using M.N.

We consider the collection of right invariant equivalence classes $[b^{2^i}]$, $i \geq 0$.

It's clear that $b^{2^i} c^i$ is in the language, but $b^{2^j} c^i$ is not when $j \neq i$

This shows that there is a separate equivalence class $[b^{2^i}]$ induced by R_L , for each $i \geq 0$.

Thus, the index of R_L is infinite and Myhill-Nerode states that L cannot be Regular. ■

1c. $\{xyz \mid x, y, z \in \{a, b\}^* \text{ and } y = xz\}$ using M.N.

We consider the collection of right invariant equivalence classes $[a^i b]$, $i \geq 0$.

It's clear that $a^i b a^i b$ is in the language, but $a^i b a^j b$ is not when $j \neq i$

This shows that there is a separate equivalence class $[a^i b]$ induced by R_L , for each $i \geq 0$.

Thus, the index of R_L is infinite and Myhill-Nerode states that L cannot be Regular. ■

Sample Assign # 5.2

2. Write a regular (right linear) grammar that generates the set of strings denoted by the regular expression $((01 + 10)^+(11))^* (00)^*$. You may use extended grammars where rules are of form $A \rightarrow \alpha$ and $A \rightarrow \alpha B$, $\alpha \in \Sigma^*$ and A, B non-terminals

$G = (\{S, T, U, V\}, \{0, 1\}, S, P)$

P:

$S \rightarrow T \mid V$

$T \rightarrow 01T \mid 10T \mid 01U \mid 10U$

$U \rightarrow 11S$

$V \rightarrow 00V \mid \lambda$

Sample Assign # 5.3

Write a Mealy finite state machine that produces the 2's complement result of subtracting 1101 from a binary input stream (assuming at least 3 bits of input)

Answer

