1. Convert the DFA below to a regular expression, first by using either the GNFA (or state ripping) or the $R_{ij}^k$ approach, and then by using regular equations. You must show all steps in each part of this solution.

![DFA Diagram]
Sample Assign # 4.1 Key (\(R_{ij}^k\))

\[
\begin{align*}
R_{11}^0 &= \lambda + 0 \quad R_{12}^0 &= 1 \quad R_{13}^0 = 0 \quad R_{14}^0 = 0 \\
R_{21}^0 &= 0 \quad R_{22}^0 &= \lambda + 1 \quad R_{23}^0 = 0 \quad R_{24}^0 = 0 \\
R_{31}^0 &= 1 \quad R_{32}^0 = 0 \quad R_{33}^0 = \lambda \quad R_{34}^0 = 0 \\
R_{41}^0 &= 0 \quad R_{42}^0 = 0 \quad R_{43}^0 = 0 \quad R_{44}^0 = \gamma + 0
\end{align*}
\]

\[
\begin{align*}
R_{11}^1 &= 0 \quad R_{12}^1 &= 0 \quad R_{13}^1 = 0 \quad R_{14}^1 = 0 \\
R_{21}^1 &= 0 \quad R_{22}^1 &= \lambda + 1 \quad R_{23}^1 = 0 \quad R_{24}^1 = 0 \\
R_{31}^1 &= 10 \quad R_{32}^1 &= 10 \quad R_{33}^1 = 10 \quad R_{34}^1 = 0 \\
R_{41}^1 &= 0 \quad R_{42}^1 = 0 \quad R_{43}^1 = 0 \quad R_{44}^1 = \lambda + 0
\end{align*}
\]

\[
\begin{align*}
R_{12}^2 &= 0 \quad R_{12}^2 &= 0 \quad R_{13}^2 = 0 \quad R_{14}^2 = 0 \\
R_{21}^2 &= 0 \quad R_{22}^2 = 1 \quad R_{23}^2 = 0 \quad R_{24}^2 = 0 \\
R_{31}^2 &= 10 \quad R_{32}^2 = 10 \quad R_{33}^2 = 10 \quad R_{34}^2 = 0 \\
R_{41}^2 &= 0 \quad R_{42}^2 = 0 \quad R_{43}^2 = 0 \quad R_{44}^2 = \lambda + 0
\end{align*}
\]

\[
\begin{align*}
R_{12}^3 &= 0 \quad R_{12}^3 &= 0 \quad R_{13}^3 = 0 \quad R_{14}^3 = 0 \\
R_{21}^3 &= 0 \quad R_{22}^3 = 0 \quad R_{23}^3 = 0 \quad R_{24}^3 = 0 \\
R_{31}^3 &= 0 \quad R_{32}^3 = 0 \quad R_{33}^3 = 0 \quad R_{34}^3 = 0 \\
R_{41}^3 &= 0 \quad R_{42}^3 = 0 \quad R_{43}^3 = 0 \quad R_{44}^3 = 0
\end{align*}
\]

\[
\begin{align*}
R_{12}^4 &= 0 \quad R_{12}^4 &= 0 \quad R_{13}^4 = 0 \quad R_{14}^4 = 0 \\
R_{21}^4 &= 0 \quad R_{22}^4 = 0 \quad R_{23}^4 = 0 \quad R_{24}^4 = 0 \\
R_{31}^4 &= 0 \quad R_{32}^4 = 0 \quad R_{33}^4 = 0 \quad R_{34}^4 = 0 \\
R_{41}^4 &= 0 \quad R_{42}^4 = 0 \quad R_{43}^4 = 0 \quad R_{44}^4 = 0
\end{align*}
\]

\[
L = R_{12}^4 + R_{13}^4
\]
Sample Assign # 4.1 Key (Rip)

\[ L = \alpha^* 1 (1 + \alpha 01^* 1 + 000^*)^* (0 + \alpha) \]
Sample Assign#4.1 Key (REQ)

A = λ + C1 + A0
B = A1 + D0 + B1
C = B0
D = C0 + D0

A = λ + C1 + A0 = λ + B01 + A0 = (λ + B01) 0*
D = C0 + D0 = B00 + D0 = B000*
B = A1 + D0 + B1 = (λ + B01) 0*1 + B0000* + B1 = 0*1+B(010*1+0000**+1)
   = 0*1(010*1+0000**+1)*
C = 0*1(010*1+0000**+1)*0
L = 0*1(010*1+0000**+1)* (0 + λ)

Consistent with Ripping, which does not always occur.
The Rijk version is really ugly
Assignment # 4.2

3. a.) Minimize the number of states in the following DFA, showing the determination of incompatible states (table on right).

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;1</td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>2,5 X</th>
<th>3,4 X</th>
<th>4,5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>✓</td>
<td>X</td>
</tr>
<tr>
<td>5</td>
<td>2,5</td>
<td>2,3 X</td>
<td>2,4</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td>2,5</td>
</tr>
</tbody>
</table>

b.) Can combine 2,4,5 and 3,6 so have states <1>, <2,4,5>, <3,6>