Assignment # 7.1 Key

1. Use the Pumping Lemma for CFLs to prove that none of the following are CFLs.
   a) L = { a^i b^j c^k d^m | m = min(max(i,j), k)}
   b) L = { a^i b^j | j = $\sum_{k=1}^{i} k$ }
   c) L = { w w^R w | w ∈ $\{a,b\}^+$ }

Assignment # 7.1a Key

1. a.) $L = \{ a^i b^j c^k d^m \mid m = \min(\max(i,j), k)\}$
   PL: Provides N>0

   We: Choose $a^N c^N d^N \in L$. As we have no b's, the # of d's is the min(#a's,#c's)

   PL: Splits $a^N c^N d^N$ into uvwxy, $|vwx| \le N$, $|vx| > 0$, such that $\forall i \ge 0\ uv^i wx^i y \in L$.

   Case 1: vx contains some a's and/or some c's (say α a's and β c's), where max(α,β)>0, and no d's. Choose i=0. Then we are decreasing the # of a's and/or # of c's while leaving the number of d's unchanged.
   The min(#a's,#c's) is N-max(α,β)<N, but we still have N d's, so this is not in L.

   Case 2: vx contains some d's (say α d's, α>0), maybe some c's, but it cannot contain any a's. Choose i=2. Then we are increasing the number of d's and maybe the number of c's while leaving the number of a's unchanged. The min(#a's,#c's) is N, but we have N+α>N d's so this is not in L.

   Cases 1 and 2 cover all possible situations, so L is not a CFL

Assignment # 7.1b Key

1. b.) **L = { $a^i\, b^j$ | j = $\sum_{k=1}^{i} k$ }**
   PL: Provides N>0

   We: Choose $a^N b^{N*(N+1)/2} \in L$

   PL: Splits $a^N b^{N*(N+1)/2}$ into uvwxy, $|vwx| \le N$, $|vx| > 0$, such that $\forall i \ge 0$ $uv^i wx^i y \in L$

   We: Choose i=2

   Case 1: vx contains only b's. But then we N a's and at least N(N+1)/2+1 b's. This is a string not in L so this case cannot be so.

   Case 2: vx contains some a's and maybe some b's. Under this circumstances uv^2wx^2y has at least N+1 a's and at most N(N+1)/2+N-1 b's. But $\sum_{k=1}^{N+1} k$ = (N+1)(N+2)/2 = N(N+1)/2 + 2N+1 + N > N*(N+1)/2+N-1 and so is not in L.*

   Cases 1 and 2 cover all possible situations, so L is not a CFL

Assignment # 7.1c Key

1. c.) **L = { w w^R w | w ∈ {a,b}$^+$ }**
   PL: Provides N>0

   We: Choose $a^N b^N b^N a^N a^N b^N = a^N b^{2N} a^{2N} b^N \in L$

   PL: Splits $a^N b^{2N} a^{2N} b^N$ *into uvwxy,* $|vwx| \leq N$, $|vx| > 0$, *such that* $\forall i \geq 0\ uv^i wx^i y \in L$

   We: Choose i=0

   Case 1: vx contains some a's, then we are decreasing the number of a's in the prefix or as part of the sequence of 2N a's. In either case the other sequence of a's is unchanged and so we either have too few a's at start or too few in second sequence. The resulting string is not in L.

   Case 2: vx contains some b's, then we are decreasing the number of b's in the suffix or as part of the sequence of 2N b's. In either case the other sequence of b's is unchanged and so we either have too few b's at end or too few in first sequence. The resulting string is not in L

   Cases 1 and 2 cover all possible situations, so L is not a CFL