COMPLEXITY
(NP)

P = { p | p IS A DECISION PROBLEM SOLVABLE IN POLYNOMIAL TIME IN TERMS OF REPRESENTATIONS OF INSTANCES OF p USING A DET. TM }

P CONTAINS DEPTH FIRST SEARCH SOLVABLE PROBLEMS — E.G., GARBAGE COLLECTION, GRAPH CONNECTIVITY, RELATIVE PRIMALITY, MEMBERSHIP IN CFL (CKY), LINEAR PROGR. OVER REALS (NOT CONSTRAINED TO INTEGER), PRIMALITY

NP = { p | p IS A DECISION PROBLEM SOLVABLE IN POLYNOMIAL TIME ... USING A NON-DET. TM }

ALSO, NP IS

NP = { p | A SOLUTION TO AN INSTANCE OF p CAN BE VERIFIED IN DET. POLY TIME }

CAN ALSO USE NOTION OF UNBOUNDED PARALLELISM

A PROBLEM IN NP IS BOOLEAN SATISFIABILITY

CAN SOLVE WITH TRUTH TABLE BY GUESSING A SOLUTION (OR CHECK A PROPOSED ASSIGNMENT OF VALUES IN LINEAR TIME)

COMPLEXITY
OTHER CLASSES

CO-NP: COMPLEMENT OF NP

NP-HARD: SUPERSET OF NP, WHEN $P \in NP, Q \in$ NP-HARD $\Rightarrow P \leq_p Q$

NP-COMPLETE: NP-HARD AND IN NP

QSAT IS NP-HARD BUT MAY NOT BE IN NP

PSPACE: SOLVABLE IN POLYNOMIAL SPACE

QSAT $\in$ PSPACE

EXP: PROBLEMS SOLVABLE IN EXPONENTIAL TIME

BIG QUESTION

$$P = NP?$$

IF $P=NP$ THEN ALL NP PROBLEMS ARE IN P

IF $P \neq NP$ THEN ALL NP-C PROBLEMS AR IN EXP BUT NOT IN P

3-SAT

$(a+\bar{b}+c)$ (exactly 3) $(- -) \cdots$

(a) $(a+b)$

$(a+a+a)$ $(a+b+a)$ $(a+b+b)$

$(t_1 + \cdots t_n)$ $n > 3$

$(a+b+c+d)$

$(a+b+z)(c+d+\bar{z})$

$(a+b+c+d+e)$

$(a+b+z)(\underbrace{c+d+e+\bar{z}}_{4})$

GRAPH THEORY PROBLEMS

UNDIRECTED ACYCLIC GRAPHS

1. COLORING PROBLEM

2. INDEPENDENT SET PROBLEM

3. VERTEX COVERING PROBLEM
   (REALLY ABOUT EDGES)

SUBSET SUM $\leq_p$ PARTITION

SUBSET SUM INSTANCE

$i_1, \ldots, i_n, G$

PARTITION INSTANCE

$i_1, \ldots, i_n, 2\Sigma - G, \Sigma + G$

$\text{TOTAL} = \Sigma + 2\Sigma - G + \Sigma + G$
$= 4\Sigma$

FOR PARTITION CONTAINING
$2\Sigma - G$ TO HAVE 2Σ
THERE MUST BE A SUBSET OF
$i_1, \ldots, i_n$ THAT EQUALS G
AND THAT LEAVES $\Sigma - G$ TO
INCLUDE IN OTHER PARTITION

$2\Sigma - G + G = 2\Sigma$
$\Sigma + G + \Sigma - G = 2\Sigma$

NOTE! $2\Sigma - G$ & $\Sigma + G$ CANNOT BE
IN SAME PARTITION AS THAT
WOULD HAVE AT LEAST 3Σ
$2\Sigma - G + \Sigma + G = 3\Sigma$

VERTEX COVERING

CHOOSE SET OF VERTICES SUCH THAT EVERY EDGE HAS AN ASSOCIATED SELECTED VERTEX

OPTIMIZATION VERSION

WHAT IS MINIMUM SIZE SET?

DECISION PROBLEM

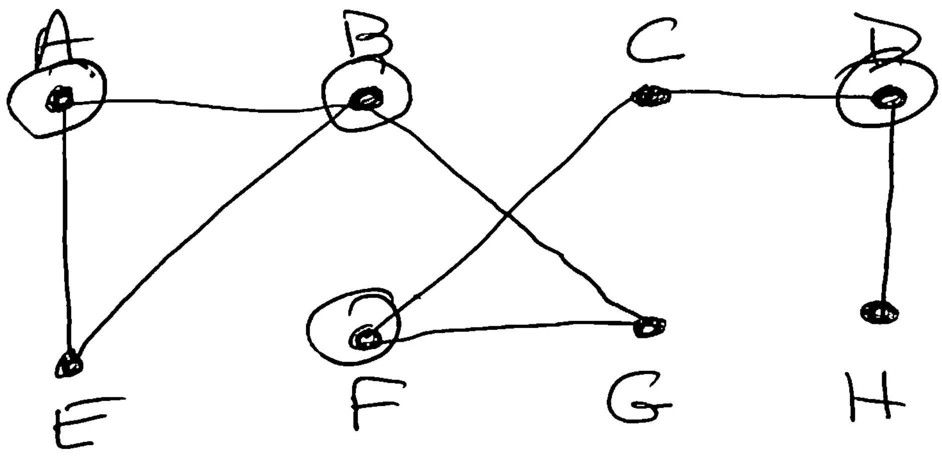
GIVEN $k > 0$, CAN THE GRAPH BE COVERED BY K VERTICES

CAN SOLVE OPTIMIZATION IN POLY TIME IF CAN SOLVE DECISION IN POLY

TRY $1, 2, \ldots, n = \#$ VERTICES

NOTE: n ALWAYS WORKS

VC K=4



A, B, F, D COVERS

OR

E, B, F, D COVERS

INDEPENDENT SET

CHOOSE SET OF VERTICES
SUCH THAT NO TWO VERTICES
HAVE COMMON EDGE (ARE CONNECTED)

OPTIMIZATION VERSION

WHAT IS MAXIMUM SIZE SET?

DECISION PROBLEM

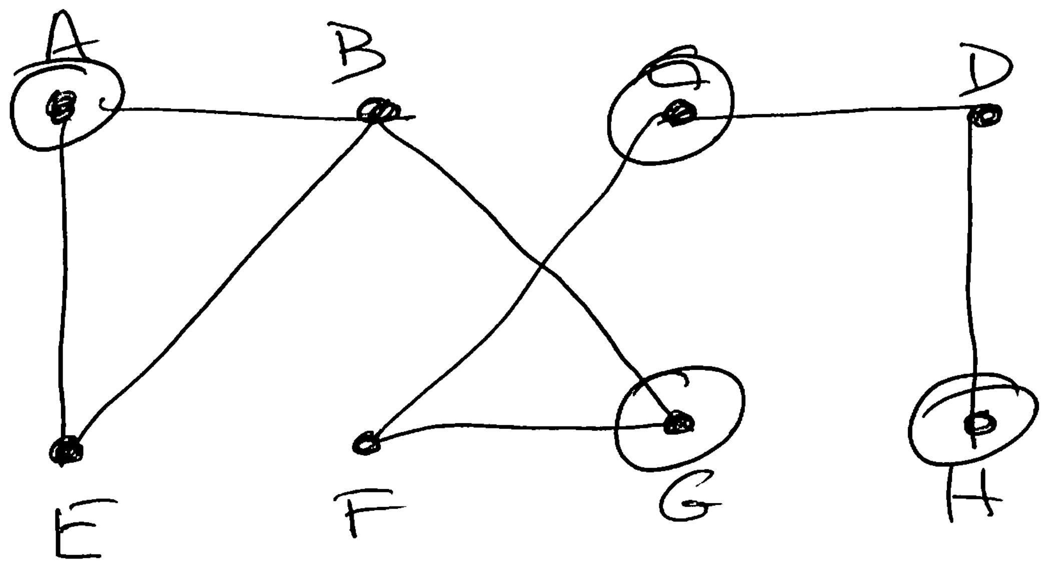
GIVEN $k > 0$, CAN THE GRAPH ~~BE~~
HAVE AN INDEPENDENT SET OF
SIZE k

CAN SOLVE OPTIMIZATION IN POLY TIME
IF CAN SOLVE DECISION IN POLY

TRY $n, n-1, \ldots, 1$ ~~#~~ n = # VERTICES

NOTE! 1 ALWAYS WORKS

IS $k=4$



A G C H is IS

OR

E G C H is IS

GRAPH COLORING

COLOR EACH VERTEX SO NO NEIGHBOR VERTEX HAS SAME COLOR

OPTIMIZATION

WHAT IS MINIMUM # OF COLORS

DECISION

GIVEN $k > 0$, CAN WE COLOR WITH k COLORS

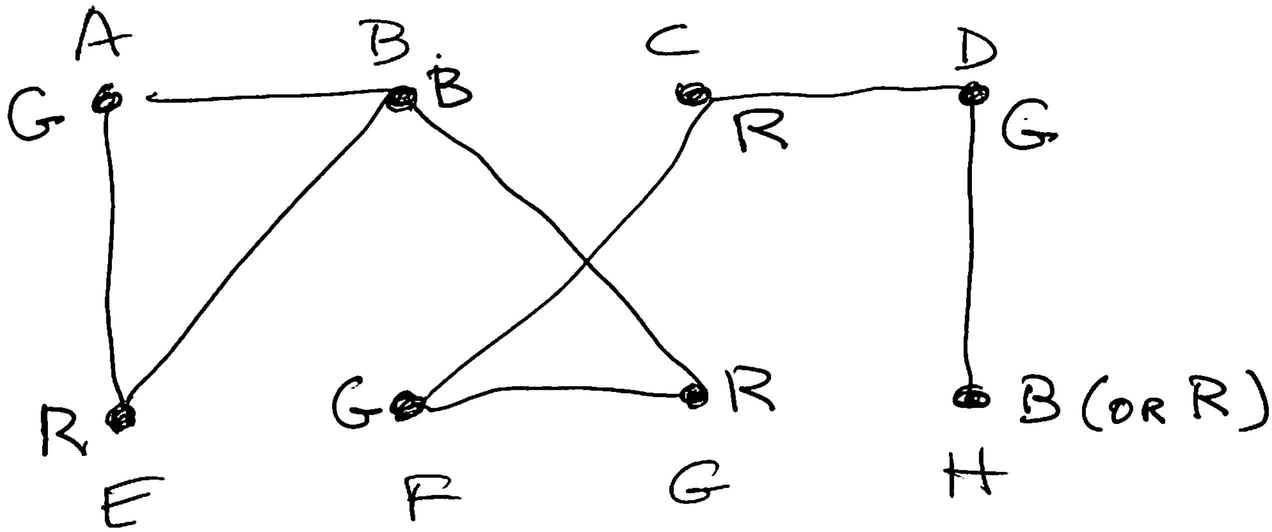
CAN SOLVE OPTIMIZATION IN POLY IF DECISION SOLVABLE IN POLY

TRY $1, 2, \ldots, n = \#$ VERTICES

n ALWAYS WORKS

4 ALWAYS WORKS IF PLANAR (NO CROSSING EDGES)

GC $k=3$



ABOVE IS ONE OF MANY 3-COLOR SOLUTIONS