$\text{SumSome} = \{ f \mid \exists x, y, z$ (DISTINCT)

WHERE $f(x)\downarrow, f(y)\downarrow, f(z)\downarrow$
AND ALL 3 ARE DISTINCT
AND $f(z) = f(x) + f(y) \}$

$\text{SumSome} = \{ f \mid \exists x, y, z$ (DISTINCT)

$\in \text{RNG}(f)$ WHERE
$z = x + y \}$

$f \in \text{SumSome} \Leftrightarrow \exists \langle x, y, z, t \rangle$

$[\text{STP}(f, x, t) \,\&\, \text{STP}(f, y, t) \,\&\, \text{STP}(f, z, t)$
$\&\, \text{VALUE}(f, z, t) = \text{VALUE}(f, x, t) +$
$\text{VALUE}(f, y, t)$

& ALL THREE VALUES ARE
DISTINCT]

MUST BE AT WORST RE

Rice's Theorem
& Sum Some

Show Non-Trivial

$I(x) = x \in \text{SumSome}$ $\quad S(x) = x+1 \in \text{SumSome}$

$C_0(x) = 0 \notin$ " $\quad \uparrow \notin \text{SumSome}$

Range Version

Let f, g be arb. $\ni \text{RNG}(f) = \text{RNG}(g)$

$f \in \text{SumSome} \iff \exists$ distinct $x, y, z \in \text{RNG}(f)$

$\ni z = x + y$

$\iff \exists$ distinct $x, y, z \in \text{RNG}(g)$

$\ni z = x + y$ as

$\text{RNG}(g) = \text{RNG}(f)$

$\iff g \in \text{SumSome}$

HALT $\leq$ SumSome

LET $\langle f, x \rangle$ BE SOME ARB. PAIR IN $\mathbb{N} \times \mathbb{N}$

FROM $\langle f, x \rangle$ DEFINE $G_{f,x}(y) = f(x) - f(x) + y$ $\forall y$

$\langle f, x \rangle \in \text{HALT} \Leftrightarrow f(x)\downarrow \Leftrightarrow$

$G_{f,x}(y) = y \quad \forall y$

$\Rightarrow G_{f,x} \in \text{SumSome}$

SINCE $G_{f,x}(1) + G_{f,x}(2) = G_{f,x}(3)$

$\langle f, x \rangle \notin \text{HALT} \Leftrightarrow f(x)\uparrow$

$\Leftrightarrow G_{f,x}(y)\uparrow \quad \forall y$

$\Rightarrow G_{f,x} \notin \text{SumSome}$

SumSome IS RE-COMPLETE

FiniteRange $= \{ f \mid \text{RNG}(f) \text{ IS FINITE} \}$

$$\exists K \; \forall \langle x, t \rangle \; [\text{STP}(f, x, t) \Rightarrow \text{VALUE}(f, x, t) \leq K]$$

$$\exists \langle J, K \rangle \; \forall \langle x, t \rangle \; [\text{STP}(f, x, t) \Rightarrow J \leq \text{VALUE}(f, x, t) \leq K]$$

RICE'S

NON-TRIVIAL

$S \notin FR$
$C_0 \in FR$

USE RANGE VERSION OF RICE'S

LET f, g BE SUCH THAT

$RNG(f) = RNG(g)$

$f \in FR \Leftrightarrow |RNG(f)|$ IS FINITE
$\Leftrightarrow |RNG(g)|$ IS FINITE
AS RANGES ARE SAME
$\Leftrightarrow g \in FR$

CLOSURE TYPE

LET A BE REC, NON-EMPTY

LET B BE RE, NON-REC

LET $C = \mathrm{MIN}(A,B) = \{z \mid x \in A, y \in B$
$z \in \min(x,y)\}$

CAN C BE REC - YES

$A = \{0\}$ $B =$ ANY RE, NON-REC SET

$C = \{0\}$

CAN C BE RE, NON-REC - YES

$A = \{2x \mid x \in \mathbb{N}\}$

$B = \{2x+1 \mid x \in HALT\}$

$C = A \cup B$ (DISCUSS WHY NON-REC)

CAN C BE NON-RE - NO

$A = RNG(f_A)$ $B = RNG(f_B)$

$C = RNG(f_C)$ WHERE

$f_C(\langle x,y \rangle) = \min(f_A(x), f_B(y))$

$$\begin{array}{ccc} B & \cap & \overline{A} \\ \nearrow & & \uparrow \\ RE & & REC \\ & & \uparrow \\ & & RE \end{array}$$

$$\mathrm{Dom}(g_B) = B \qquad \mathrm{Dom}(g_{\overline{A}}) = \overline{A}$$

$$\mathrm{Dom}(g_C) = B \cap \overline{A}$$

$$x \in B \iff \exists t \ STP(g_B, x, t)$$

$$x \in \overline{A} \iff \exists t \ STP(g_{\overline{A}}, x, t)$$

$$x \in B \cap \overline{A} \iff \exists t \ STP(g_B, x, t) \,\&\, ST(g_{\overline{A}}, x, t)$$

LET A BE REC, NON-EMPTY

B BE RE, NON-REC

LET $C = \text{MIN}(A, B) =$

$$\{z \mid x \in A, y \in B \ \& \ z = \text{MIN}(x, y)\}$$

CAN C BE REC - YES

$A = \{0\}$

$C = \text{MIN}(\{0\}, B) = \{0\}$ SINCE $B \neq \emptyset$

CAN C BE RE, NON-REC - YES

$A = \{2x \mid x \in \mathbb{N}\}$

$B = \{2x+1 \mid x \in \text{HALT}\}$

$C = A \cup B$

CAN C BE NON RE - NO

$$f_C(\langle x, y \rangle) = \underset{\text{MIN}}{\vee}(f_A(x), f_B(y))$$

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$D = \sim C$ C NON-RE

C COULD BE $\overline{HALT}$ $\forall t\,[STP(f,x,t)]$

$\sim C$ IS HALT

C COULD BE TOTAL $\forall\exists$

$\exists\forall$

$\sim C$

$D \subseteq \mathbb{N}$ ALL ~~ARE~~ POSSIBLE

$D = \sim B$ B RE, NONREC

D IS A $\forall$ SO NOT RE

$D = B - A$ B RE, NON REC, A REC

$A = \mathbb{N}$ THEN $D = \emptyset$ REC

$A = \emptyset$ THEN $D = B$

CANNOT BE NR (WHY?)