PCP to Ambiguity
Explicit Example

$n = 3$, $\Sigma = \{a, b\}$ $X = (aba, bb, a)$ $Y = (bab, b, baa)$

$G = (\{S, A, B\}, \Sigma, R, S)$ $\cup\ [i]$

$1 \le i \le n$

R:

$S \to A \mid B$

$A \to aba\,A[1] \mid aba[1]$

$\quad \mid bb\,A[2] \mid bb[2]$

$\quad \mid a\,A[3] \mid a[3]$

$B \to bab\,B[1] \mid bab[1]$

$\quad \mid b\,B[2] \mid b[2]$

$\quad \mid baa\,B[3] \mid baa[3]$

$A \underset{2}{\Rightarrow} bbA[2] \underset{3}{\Rightarrow} bbaA[3][2]$

$\underset{1}{\Rightarrow} bbaabaA[1][3][2]$

$\underset{2}{\Rightarrow} bbaababb[2][1][3][2]$

$B \underset{2}{\Rightarrow} bB[2] \underset{3}{\Rightarrow} bbaaB[3][2]$

$\underset{1}{\Rightarrow} bbaababB[1][3][2]$

$\underset{2}{\Rightarrow} bbaababb[2][1][3][2]$

AMBIGUOUS

NON-EMPTINESS OF CSL

(1) SINCE $L_1 \cap L_2$ IS A CSL WHEN
L_1, L_2 ARE CFLs THEN
$L_1 \cap L_2 \neq \phi \leq$ CSL NON-EMPTY

SO NON-EMPTINESS OF CSL UNDEC

(2) DIRECT PCP $\leq$ CSL NON-EMPTY

$G = (\{S, T\} \cup \Sigma, \{*\}, R, S)$

R:

$S \rightarrow x_i S y_i^R \mid x_i T y_i^R \quad 1 \leq i \leq n$

$\forall a \in \Sigma$

$aTa \rightarrow * T *$

$* a \rightarrow a *$

$a * \rightarrow * a$

$T \rightarrow *$

$L(G) \neq \phi$ IFF PCP HAS SOLN.

ALSO $L(G)$ INF. IFF PCP HAS SOLN,

ALSO $L(G) = \phi$ IFF PCP HAS NO SOLN,

ALL ARE THUS UNDEC.

NON-EMPTY INTERSECTION

CONSIDER A-RULES + B-RULES

$G_A = (\{A\}, \Sigma, R_A, A)$

$G_B = (\{B\}, \Sigma, R_B, B)$

$L(G_A) \cap L(G_B) \neq \emptyset$

IFF $L(G)$ AMBIGUOUS

IFF SOLUTION TO INSTANCE OF PCP

THUS

$L(G)$ AMBIGUOUS IS UNDEC.

PCP $\leq$ AMB. OF CFL

AND

$L(G_A) \cap L(G_B) \neq \emptyset$ UNDEC

PCP $\leq L_1 \cap L_2 \neq \emptyset$ L_1, L_2 CFLs

PCP UNSOLVABLE (SKETCH)

SEMI-THUE IS REWRITING SYSTEM OF

FOR $\alpha_i \Rightarrow \beta_i$ $\alpha_i, \beta_i \in \Sigma^*$

$1 \le i \le n$ RULES

CAN SHOW

HALT $\le$ $WORD_{ST}$

WHERE $WORD_{ST}$ IS PROBLEM FOR TWO ARB. STRINGS $w_1, w_2 \in \Sigma^*$

DOES $w_1 \overset{*}{\Rightarrow} w_2$

CAN THEN SHOW

$WORD_{ST} \le PCP$

AND SO PCP IS UNSOLVABLE

EXPLICIT PCP EXAMPLE

$n = 3,\ \Sigma = \{a, b\}$

$X = (aba, bb, a)$ $\quad Y = (bab, b, baa)$

MUST START WITH 2 AS ONLY ONE WHERE FIRST LETTERS MATCH

SOLUTION IS 2, 3, 1, 2

X	bb	a	aba	bb
Y	b	baa	bab	b

IF ONE SOLUTION, INFINITE NUMBER OF SOLUTIONS

TRACES & NON-TRACES

CONSIDER A MODEL OF COMPUTATION, M, AND ITS IDs .... $X_1, X_2, \ldots, X_k, \ldots$

A TRACE IS A SEQUENCE OF IDS

$X_1 \# X_2 \# \cdots \# X_n$

WHERE $X_i \underset{M}{\Rightarrow} X_{i+1}$ $\quad 1 \le i < n$

A TRACE IS TERMINAL IF X_n IS A HALTING ID.

TRACES ARE CSLs

THE COMPLEMENTS OF TRACES ARE CFLs

IF M HAS NO TERMINATING TRACES THEN THE COMPLEMENT OF TERMINATING TRACES IS A CFL $= \Sigma^*$

THIS IS A SKETCH AS TO WHY $L(G) = \Sigma^*$ IS UNDEC. FOR G A CFL (OR WORSE)

NOTE: ALSO SHOWS $L(G) = \emptyset$ IS UNDEC. FOR G A CSL