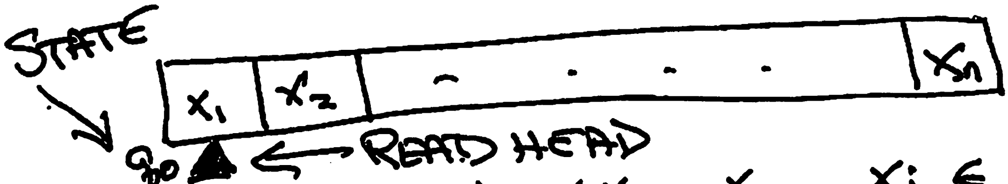
WEEK #1

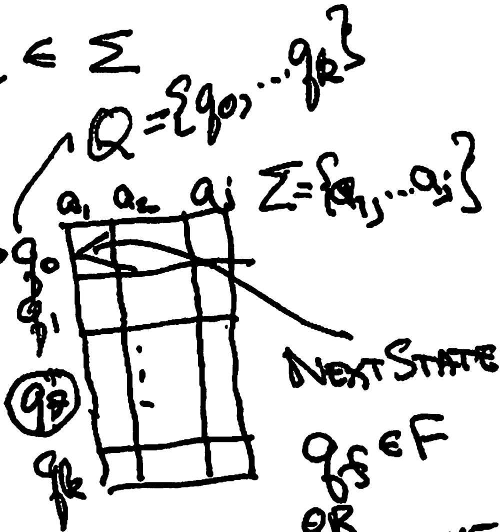
①

FINITE STATE AUTOMATON
DFA vs NFA



INPUT IS $w = x_1 x_2 \cdots x_n$, $x_i \in \Sigma$

$q_0 \in Q$ IS START STATE



ACTIONS ARE:

- LOOK UP IN STATE TABLE
  ENTITY IS AT ROW q_m COL a_i
  WHERE STATE IS q_m
  INPUT SYMBOL IS a_i
- CHANGE STATE TO ENTITY
  AT q_m, a_i
- MOVE READ HEAD ONE CELL ON TAPE (RIGHT)
- IF FALL OFF END THEN DONE
- IF IN ACCEPTING STATE, $p \in F \subseteq Q$
  ACCEPT IF IN ACCEPTING STATE, OTHERWISE
  REJECT OTHERWISE
- IF NOT OFF END (MORE TO READ),
  ITERATE PROCESS

DFA HAS ONE STATE FOR EACH PAIR q_m, a_i

MAPPING IS $\delta : Q \times \Sigma \to Q$

PROCESS ALWAYS HALTS WITH ACCEPT OR REJECT

WEEK #1

FORMAL MODEL OF DFA

$A = (Q, \Sigma, \delta, q_0, F)$

Q IS FINITE SET OF STATES

Σ IS FINITE ALPHABET FOR INPUT STRINGS

δ IS TRANSITION (NEXT STATE) FUNCTION

$\delta : Q \times \Sigma \to Q$

$q_0 \in Q$ IS START STATE

$F \subseteq Q$ IS SET OF FINAL (ACCEPTING) STATES

TRANSITIONS AND LANGUAGE

$\delta^*: Q \times \Sigma^* \to Q$ IS REFLEXIVE, TRANSITIVE CLOSURE OF δ

$\delta^*(q, \lambda) = q$

$\delta^*(q, ax) = \delta^*(\delta(q, a), x)$ $a \in \Sigma, x \in \Sigma^*$

NOTE: $\delta^*(q, a) = \delta(q, a)$

$\delta^+(q, w) = \delta^*(q, w)$ WHEN $|w| > 0$

IF $A = (Q, \Sigma, \delta, q_0, F)$ THEN

$\mathcal{L}(A) = \{ w \mid \delta^*(q_0, w) \in F \}$

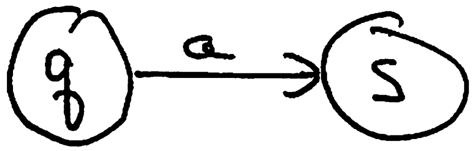
$\mathcal{L}(A)$ IS CALLED A FINITE STATE OR A REGULAR LANGUAGE

THE CLASS OF REGULAR LANGUAGES IS DEFINED AS THE SET OF ALL LANGUAGES ACCEPTED BY DFAs.

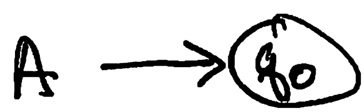
TRANSTION TABLES VS STATE DIAGRAMS

TRANSITION TABLES ARE GREAT FOR COMPUTERS AND AWFUL FOR HUMANS

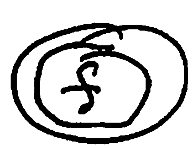
STATE DIAGRAMS ARE ROOTED GRAPHS THAT HAVE DIRECTED ARCS FROM STATE q TO STATE s, LABELLED a, WHENEVER $\delta(q, a) = s$



THE ROOT IS THE START STATE AND MUST HAVE AN ENTERING ARC WITH NO PRIOR NODE (CAN THINK OF ITS BEING LABELED WITH λ



ALL FINAL STATES HAVE A DOUBLE CIRCLE



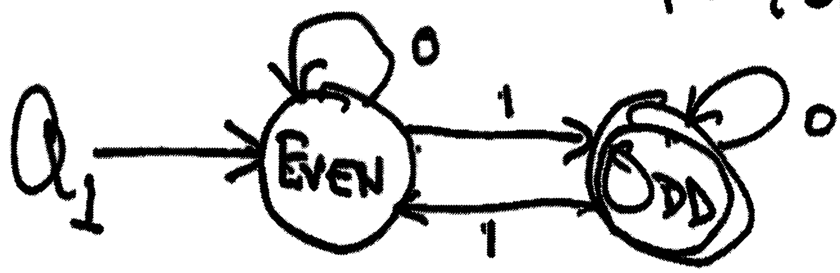
WEEK #1

SAMPLES

ODD PARITY

$\Sigma = \{0,1\}$ $Q = \{\text{Even}, \text{Odd}\}$

$F = \{\text{Odd}\}$ $q_0 = \text{Even}$



	0	1
→ E	E	Ø
Ø	Ø	E

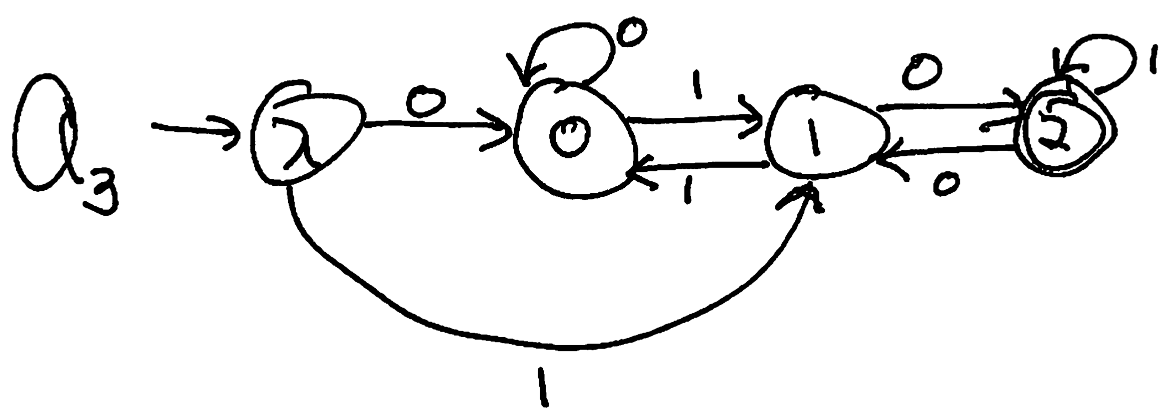
WEEK #1

ANOTHER SAMPLE

ACCEPT BINARY STRING (MSB TO LSB) THAT, WHEN INTERPRETED AS DECIMAL NUMBERS, ARE 2 MOD 3

$Q = \{\lambda, 0, 1, 2\}$ $q_0 = \lambda$ $F = \{2\}$

$\Sigma = \{0, 1\}$



NOTE: $2*0+0=0$

$2*0+1=1$

$2*1+0=2$

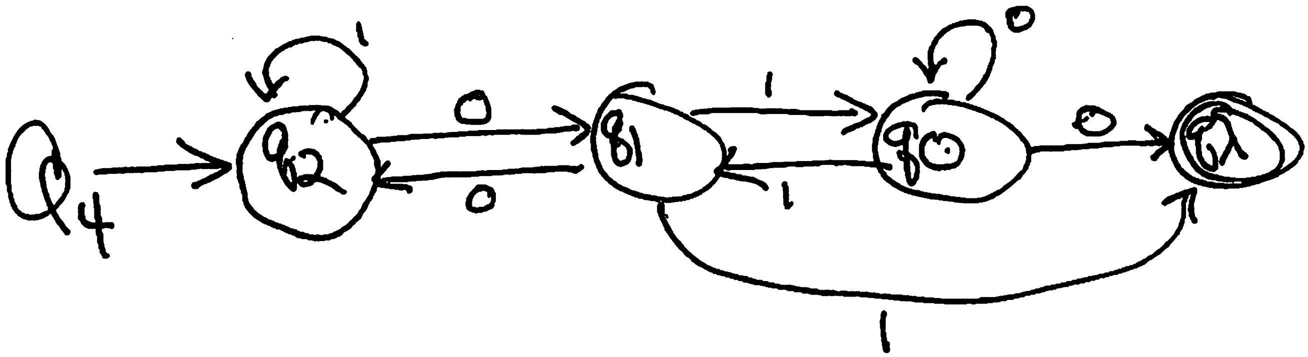
$2*1+1=3=0 \text{ MOD } 3$

$2*2+0=4=1 \text{ MOD } 3$

$2*2+1=5=2 \text{ MOD } 3$

WEEK #1

SAMPLE Q_3 READ LSB TO MSB



THIS REVERSES EARLIER MACHINE

(WILL SHOW REVERSE OF REGULAR IS REGULAR IN A BIT)

BUT NOW WE ARE NON-DETERMINTIC

$$\delta(q_1,1) = \{q_0 q_\lambda\}$$
$$\delta(q_\lambda,0) = \delta(q_\lambda,1) = \{\}$$
$$\delta(q_0,0) = \{q_0, q_\lambda\}$$

TWO CASES OF NON-DET.

ONE CASE OF NO TRANSITION

THIS IS AN NFA NOT A DFA

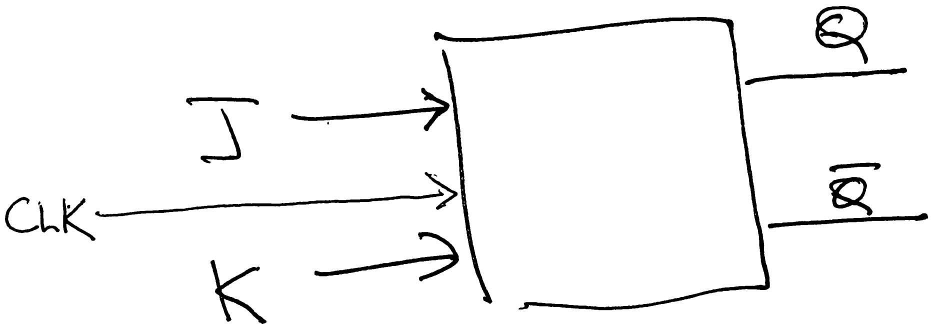
WEEK #1

WHERE DID (DFA) FSA COME FROM?

MATHEMATICAL/READABLE DESCIPTION OF SEQUENTIAL CIRCUITS

BOOLEAN + FLIP/FLOPS (2 VALUES)

PARITY CHECKER HAS JUST ONE FLIP/FLOP



$J=0, K=0$ NO CHANGE

$J=1, K=0$ $Q=1, \overline{Q}=0$

$J=0, K=1$ $Q=0, \overline{Q}=1$

$J=1, K=1$ FLIP VALUES

ODD PARITY JUST STARTS AT 0 AND EACH BIT IS REPLICATED ON J AND K. 1'S FLIP STATE; 0'S MAKE NO CHANGE

WEEK #1

WHERE ARE FSAs USED?

REDUCING STATES IN SEQUENTIAL CIRCUITS

PATTERN MATCHING

- GREP & ITS VARIANTS
- PASSWORD LEGALITY
- DATA VALIDITY IN GENERAL
- LEXICAL ANAYSIS

GAME LOGIC

1

OR

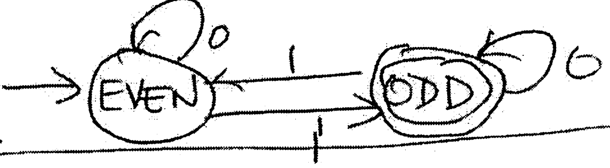
9

WEEKS #2 + #1
SAMPLES

ODD PARITY $\Sigma = \{0,1\}$ $Q = \{EVEN, ODD\}$

$F = \{ODD\}$ $q_0 = EVEN$

$Q_1 \rightarrow$



CHECK FOR 2'S COMPLEMENT

$\Sigma = \{0,1\} \times \{0,1\} = \{00, 01, 10, 11\}$

THINK OF THIS AS TWO SYCHRONIZED CHANNELS OF INPUT (TOP IS #, BOTTOM IS 2'S COMPLEMENT)

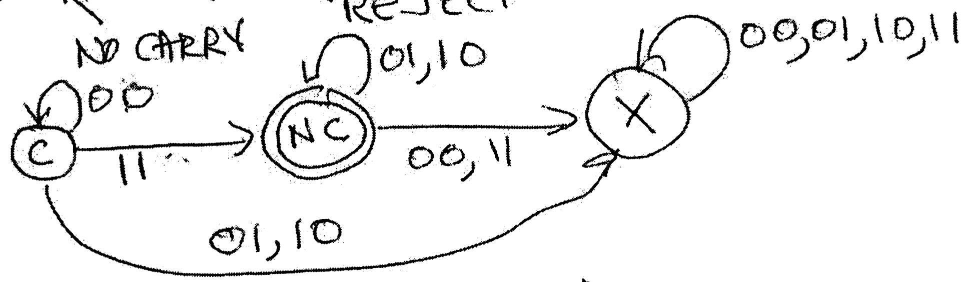
WE WILL DO LEAST SIGNIFICANT TO MOST SIGNIFICANT BIT (LSB TO MSB) FIRST

$q_0 = C$ $F = \{NC\}$

$Q = \{C, NC, X\}$ ← REJECT

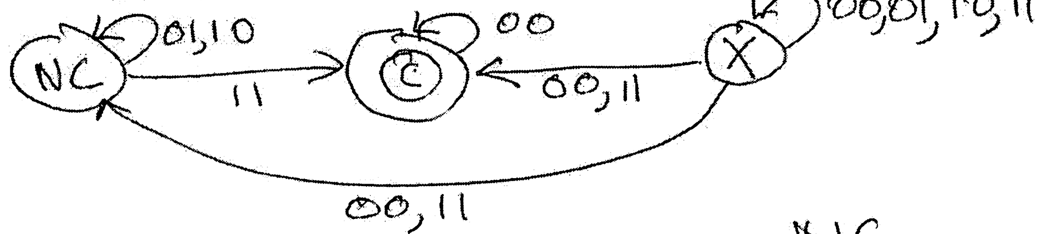
CARRY NO CARRY

$Q_2 \rightarrow$



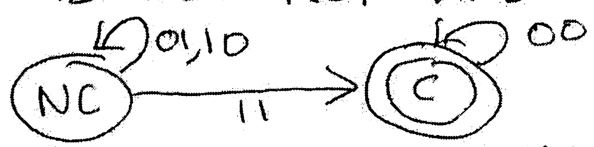
MSB TO LSB (FROM Q_2)

$Q_3 \rightarrow$

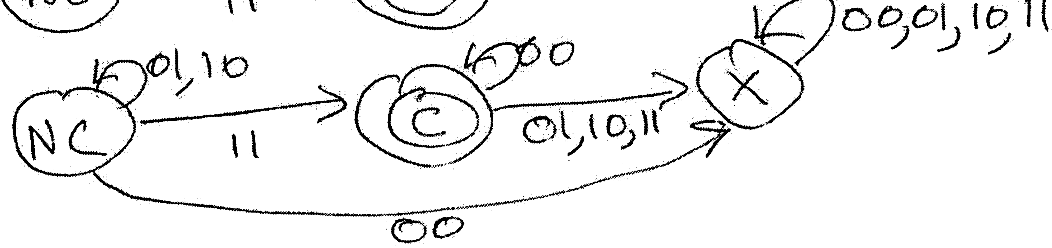


BUT X IS NOT REACHABLE FOM NC

$Q_3' \rightarrow$



OR $Q_3'' \rightarrow$



CLOSURE PROPERTIES

COMPLEMENT OF A DFA

$$Q = (Q, \Sigma, \delta, q_0, F)$$

IS JUST

$$\overline{Q} = (Q, \Sigma, \delta, q_0, Q-F)$$

THE LANGUAGE OF Q IS

$$\mathcal{L}(Q) = \{w \mid \delta(q_0, w) \in F\}$$

THE LANGUAGE OF $\overline{Q}$ IS

$$\mathcal{L}(\overline{Q}) = \{w \mid \overline{\delta}(q_0, w) \in Q-F\}$$

NOTE: $\overline{Q}$ IS STILL A DFA

WHAT ABOUT UNION

$Q_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$

$Q_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$

WHERE $Q_1 \cap Q_2 = \phi$

NO OVERLAPPING STATE LABELS

IF ALLOWED λ-TRANSITIONS

(TRANSITIONS THAT READ NO INPUT)

COULD DO

$Q_3 = (Q_1 \cup Q_2 \cup \{q_3\}, \delta_3, q_3, F_1 \cup F_2)$

WHERE $q_3 \notin (Q_1 \cup Q_2)$

$\delta_3(q, a) = \delta_1(q, a)$ WHENEVER $a \in \Sigma, q \in Q_1$

$\delta_3(q, a) = \delta_2(q, a)$ WHENEVER $a \in \Sigma, q \in Q_2$

$\delta_3(q_3, \lambda) = \{q_1, q_2\}$

NON-DETERMINISTIC

BOOLEAN SET OPS

NICE META CONSTRUCTION

$$\mathcal{A}_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$$

$$\mathcal{A}_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$$

$$Q_3 = Q_1 \times Q_2 = \{\langle q, p\rangle \mid q \in Q_1, p \in Q_2\}$$

$$\mathcal{A}_3 = (Q_3, \Sigma, \delta_3, \langle q_1, q_2\rangle, F_3)$$

$$\delta_3(\langle p, q\rangle, a) = \langle \delta_1(p, a), \delta_2(q, a)\rangle$$

$$F_3 = F_1 \times F_2 \qquad \cap$$

$$F_3 = F_1 \times Q_2 \cup Q_1 \times F_2 \qquad \cup$$

$$F_3 = F_1 \times Q_2 \cup Q_1 \times F_2 - F_1 \times F_2 \qquad \oplus$$

$$F_3 = F_1 \times (Q_2 - F_2) \cup (Q_1 - F_1) \times F_2 \qquad \oplus$$