**COT 4210 Fall 2018** **Midterm 2 Sample** **Name: ______________**

**Total Points Available 67** **Your Raw Score _______** **Grade:____________**

*2* **1.** Let $\mathbf{A} = (\{q1, \ldots q10\}, \{0,1\}, q1, \{q7\})$ be some DFA. Assume you have computed the sets, $\mathbf{R^k_{i,j}}$, for $\mathbf{0 \le k \le 9, 1 \le i \le 10, 1, 1 \le j \le 10}$. How do you compute $\mathbf{L(A) = R^{10}_{1,7}}$, based on the previously computed values of the $\mathbf{R^k_{i,j}}$'s?

*4* **2.** Write a Context Free Grammar for the language **L**, where
$\mathbf{L = \{ a^i\, b^j\, c^k \mid i \le (j + k) \}}$

*5* **3.** Assume **A** and **B** are arbitrary Context Free languages. Indicate, for each of the following operations, whether the language **L** is guaranteed to be Context Free (Note: Regular languages are Context Free). No proofs or examples are required.

| **Operation** | **Is L guaranteed to be a CFL? (Y or N)** |
|---|---|
| **L ⊂ A (Subset)** | |
| **L = A ∩ B (Intersection)** | |
| **L = A • B (Concatenation)** | |
| **L = A ⊕ B ({ x \| x is in either A or B, but not both }** | |
| **L = Max(A) ({ x \| x∈A but no xy∈A, \|y\|>0 }** | |

***8*** **4.** Use the Pumping Lemma for CFLs to show that the following language **L** is not Context Free.
**L = { $a^n b^{2^n}$ | n>0 }**. Be explicit as to why each case you analyze fails to be an instance of **L**.
I will do the first two steps for you.

***ME: Assume L is Context Free***
***PL: Provides a whole number N>0 that is the value associated with L based on the Pumping Lemma***

*6* **5.** Consider some languages **A** and **B** that are both Context Free, and neither is Regular. Define **L = A ∪ B**. Give explicit examples of languages **A** and **B**, and explicitly describe **L**, or argue that this is impossible based on some well-known result, for each of the following.

a.) **L** is Regular

b.) **L** is Context Free, non-Regular.

c.) **L** is Context Sensitive, non-Context-Free.

***10*** **6.** Present the **CKY** recognition matrix for the string **abbccc** assuming the Chomsky Normal Form grammar, **G = ({S, A, B, C, X, Y, Z }, {a,b,c}, R, S)**, specified by the rules **R**: Note: **abbccc** is in **L(G)** so that should help you if you make an error and don't see **S** at bottom of matrix.

**S → AB**
**A → XA | a**
**B → CZ | BZ | b | c**
**C → YB**
**X → a**
**Y → b**
**Z → c**

| | **a** | **b** | **b** | **c** | **c** | **c** |
|---|---|---|---|---|---|---|
| **1** | | | | | | |
| **2** | | | | | | |
| **3** | | | | | | |
| **4** | | | | | | |
| **5** | | | | | | |
| **6** | | | | | | |

**A little help from your friends**

| **Non-Terminal** | **First Symbol in Rules** | **Second Symbol in Rules** |
|---|---|---|
| **S** | None | None |
| **A** | **S → AB** | **A → XA** |
| **B** | **B → BZ** | **S → AB; C → YB** |
| **C** | **B → CZ** | None |
| **X** | **A → XA** | None |
| **Y** | **C → YB** | None |
| **Z** | None | **B → BZ; B → CZ** |

***8*** 7. Prove that Context-Free Languages are closed under **div3** where **L** is a CFL over the alphabet **Σ**, and **div3(L) = { x | xy ∈ L** and **|x| modulo 3 = 0** and **|y| ∈ {0,1,2} }.**
In words, we remove as few characters as needed from the end of a string in **L**, so the resulting string's length is a multiple of **3**.
You may assume substitution **f(a) = {a, a'},** and homomorphisms **g(a) = a'** and **h(a) = a, h(a') = λ**.
Here **a∈Σ** and **a'** is a distinct new character associated with each **a∈Σ**.
You must be very explicit, describing what is produced by each transformation you apply and what kind of language results.

*12* **8.** Consider the CFG **G = ( { S, A, B } , { a , b } , R , S )** where **R** is:

**S → AB**
**A → aA | a**
**B → aBb | ab**

In the PDAs below, you may (and are encouraged) to use a transition diagram where transitions have arcs with labels of form **a, α → β** where **a** ∈ Σ∪{λ}, α, β ∈ Γ*. Note: This just means that you can use extended stack operations that push more than one symbol onto stack.

**a.)** Present a pushdown automaton that parses the language L(G) using a top down strategy.
INITIAL CONTENTS OF STACK = ______

**b.)** Now, using the notation of **ID**s (Instantaneous Descriptions, **[q, x, z]**), describe how your PDA in (a) accepts strings generated by **G**.

**c.)** Present a pushdown automaton that parses the language **L(G)** using a bottom up strategy. Note: I am fine with your showing strings that are on top of the stack in either reversed or non-reversed form.
INITIAL CONTENTS OF STACK = ______

**d)** Now, using the notation of **ID**s (Instantaneous Descriptions, **[q, x, z]**), describe how your PDA in (c) accepts strings generated by **G**.

**9.** Consider the context-free grammar **G = ( { S, A, B, C, D }, { a,b,c }, R, S )**, where **R** is:

**S → BC | AC | ABC**
**A → aA | λ**
**B → ABb | Bb**
**C → bCc | bc**
**D → bBc | Dc | λ**

*3* **a.)** Remove λ-rules, creating an equivalent grammar **G'**. Show all rules. ***Nullable*** *= { }*

*2* **b.)** Remove all **unit** rules', creating an equivalent grammar **G''**. Show all rules.
***Unit(S) = { }; Unit(A) = { }; Unit(B) = { }; Unit (C) = { }; Unit(D) = { }***

*2* **c.)** Remove all unproductive symbols, creating an equivalent grammar **G'''**. Show all rules.
***Productive*** *= { };* ***Unproductive*** *= { }*

*2* **d.)** Remove all unreachable symbols, creating an equivalent grammar $\mathbf{G^{iv}}$. Show all rules.
***Unreachable*** *= { }*

*3* **e.)** Convert grammar $\mathbf{G^{iv}}$ to its **Chomsky Normal Form** equivalent, $\mathbf{G^{V}}$. Show all rules.