

- 4 1. Present the transition diagram for a DFA that accepts the set of binary strings that represent the magnitude of numbers that have a remainder of 1, when divided by 3. Numbers are read most to least significant digit, so 01 (1), 111 (7) and 10011 (19) are accepted, but 0000 (0), 110 (6) and 010001 (17) are not. Note: Leading zeroes are allowed.



- 5 2. Consider the following assertion:

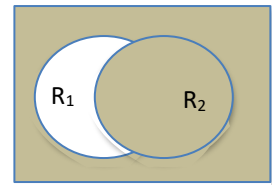
Let  $R_1$  and  $R_2$  be regular languages that are recognized by  $A_1$  and  $A_2$ , respectively, where

$A_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$  and  $A_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$  are DFAs.

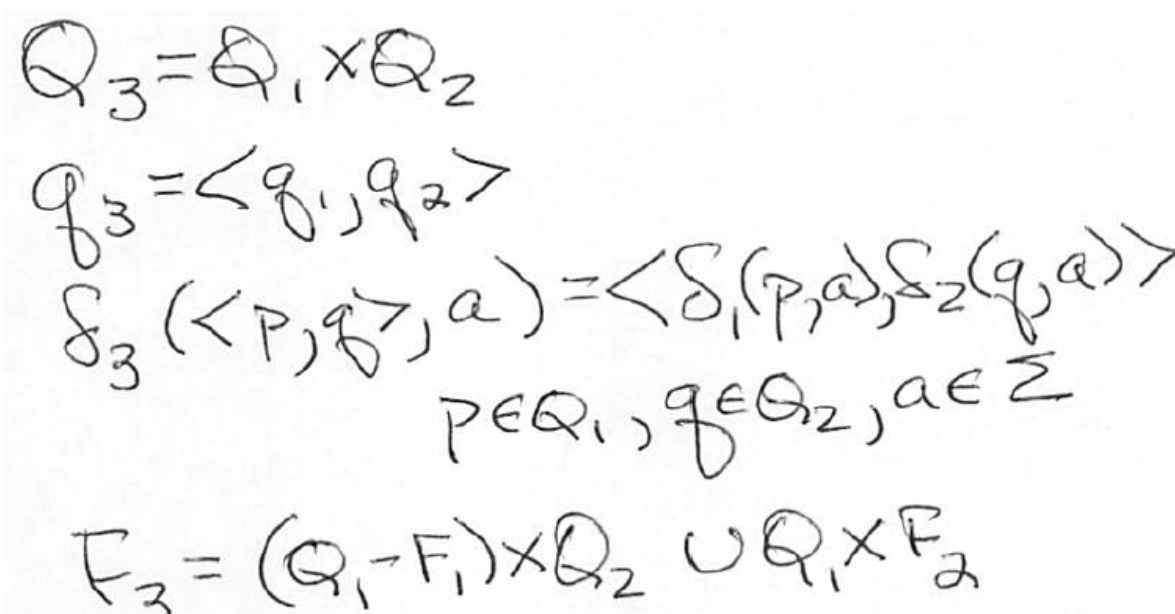
Show that  $L = \sim(R_1 - R_2) = \{w \mid w \text{ is in the complement of } (R_1 - R_2)\} =$

$\sim(R_1 \cap \sim R_2) = \sim R_1 \cup R_2$

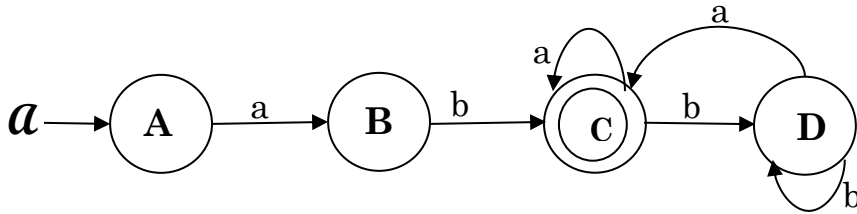
is also regular, where  $\sim$  means set complement and  $-$  means set difference.



Present a DFA construction  $A_3 = (Q_3, \Sigma, \delta_3, q_3, F_3)$ , where  $L(A_3) = \sim(R_1 - R_2)$ . You must clearly define  $Q_3, \delta_3, q_3,$  and  $F_3$ . However, I do not require you to prove or even justify your choice of final set, just to get it right.



3. Let  $L$  be defined as the language accepted by the finite state automaton  $\mathcal{A}$ :



- 7 a.) Present the regular equations associated with each of  $\mathcal{A}$ 's states, solving for the regular expression associated with the language recognized by  $\mathcal{A}$ .

$$\begin{aligned}
 A &= \lambda & B &= Aa = a \\
 C &= Bb + Da + Ca = ab + Da + Ca \\
 D &= Cb + Db = Cb^+ \\
 C &= ab + Cb^+a + Ca \\
 &= ab(b^+a + a)^* = ab(b^*a)^*
 \end{aligned}$$

- 5 b.) Assuming that we designate  $A$  as state 1,  $B$  as state 2,  $C$  as state 3 and  $D$  as state 4. Kleene's Theorem allows us to associate regular expressions  $R_{ij}^k$  with  $\mathcal{A}$ , where  $i \in \{1..4\}$ ,  $j \in \{1..4\}$ , and  $k \in \{0..4\}$ . The following are values of

$$R_{1,3}^3 = aba^* , R_{2,3}^3 = ba^* , R_{3,3}^3 = a^* , R_{3,4}^3 = b$$

What are the values of the following?

$$R_{4,3}^3 = a^+ , R_{4,4}^3 = a^+b + b = a^*b$$

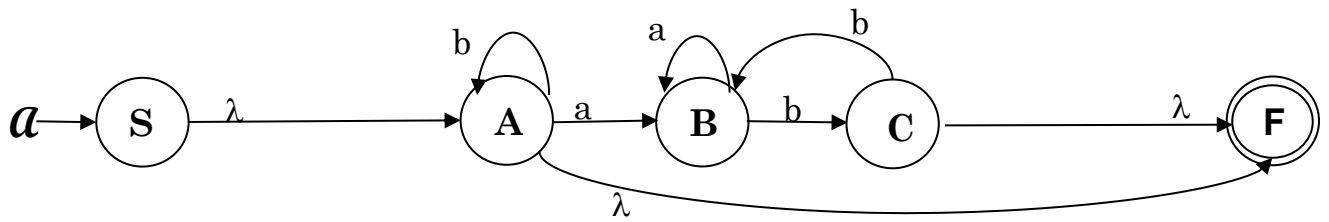
How is  $R_{3,3}^4$  calculated from the set of  $R_{ij}^3$ 's above? Give this abstractly in terms of the  $R_{ij}^3$ 's

$$R_{33}^4 = R_{33}^3 + R_{34}^3 \tilde{R}_{44}^{3*} R_{43}^3$$

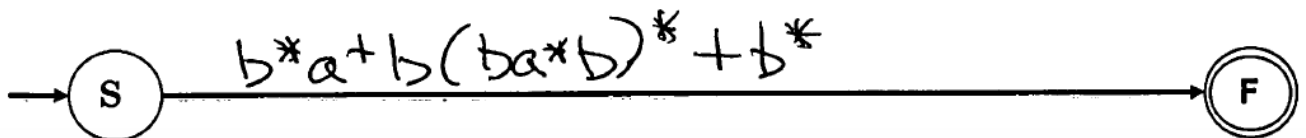
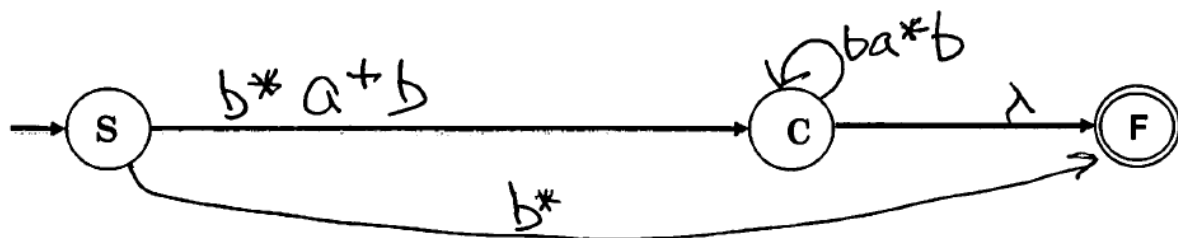
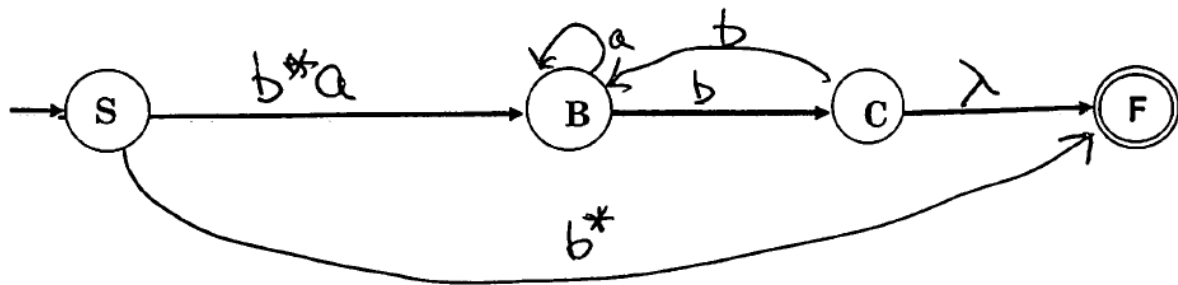
What expression does  $R_{3,3}^4$  evaluate to, given that you have all the component values?

$$R_{33}^4 = a^* + b(a^*b)^*a^+$$

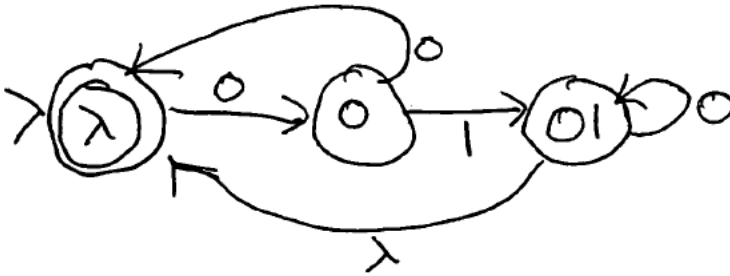
6 4. Let  $L$  be defined as the language accepted by the NFA  $\mathcal{a}$ :



Using the technique of replacing transition letters by regular expressions and then ripping states from a GNFA to create new expressions, develop the regular expression associated with the automaton  $\mathcal{a}$  that generates  $L$ . I have included the states of GNFA's associated with removing states  $A, B$  and then  $C$ , in that order. You must use this approach of collapsing one state at a time, showing the resulting transitions with non-empty regular expressions.



- 3 5. Consider the regular expression  $R = (00 + 010^*)^*$   
 The only + used here stands for union. Show an NFA (do by transition diagram) that accepts  $R$ .



- 4 6.  $OddLetters(L) = \{ x_1 x_3 \dots x_{2n+1} \mid \text{where } x_1 x_2 x_3 \dots x_{2n} x_{2n+1} \in L \text{ or } x_1 x_2 x_3 \dots x_{2n} x_{2n+1} x_{2n+2} \in L \}$ ,  
 where each  $x_i \in \Sigma$

Show that any class of languages (in particular the Regular Languages) that is closed under substitution, concatenation and intersection with Regular Languages is also closed under **OddLetters**. A constructive solution is all I ask; no proof required. I'll help. Consider using substitutions  $f(a) = \{a, a'\}$ ;  $g(a) = a'$ ;  $h(a) = a, h(a') = \lambda$ , where  $a \in \Sigma$  and  $a'$  is a new symbol uniquely associated with the symbol  $a$ .

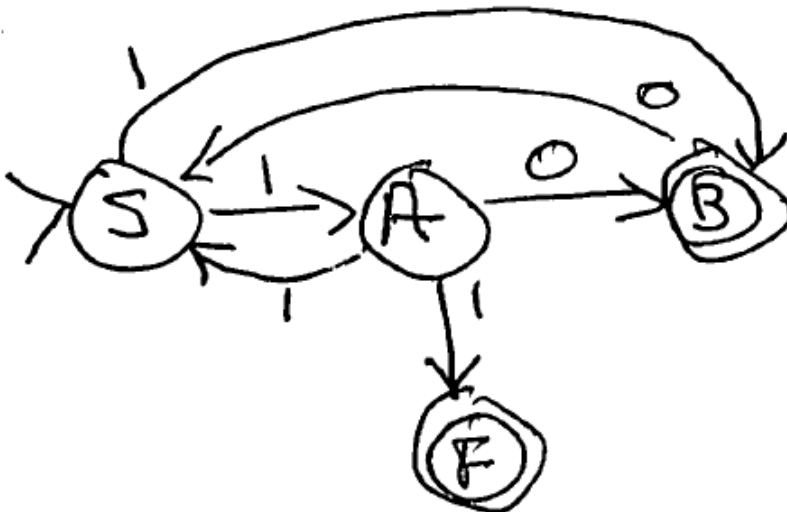
$$OddLetters(L) = h(f(L) \cap (\Sigma \cdot g(\Sigma))^* (\Sigma \cup \Sigma \cdot g(\Sigma)))$$

- 4 7. Consider the regular grammar  $G = (\{S, A, B\}, \{0, 1\}, S, P)$  where  $P$  is the set of rules:

$S \rightarrow 1B \mid 1A$   
 $A \rightarrow 0B \mid 1S \mid 1$   
 $B \rightarrow 0S \mid \lambda$

Present an NFA  $\mathcal{A}$  that accepts the language generated by  $G$ :

**NOT ON EXAM**



- 6 8. Apply the Pumping Lemma to show the following is **NOT** regular. Be sure to differentiate the steps (contributions) to the process provided by the Pumping Lemma and those provided by you. Be sure to be clear about the contradiction. I'll even start the process for you.

$$L = \{ a^i b^j \mid i < j \}$$

*PL: Gives you a value of  $N > 0$ .*

*You: Choose an element in  $L$  whose length is at least  $N$  (now you need to choose that element)*

$$\text{ME: } a^N b^{N+1} \in L$$

$$\text{PL: } xyz = a^N b^{N+1}, |xy| \leq N, |y| > 0$$

$$\forall i, xy^i z \in L, i \geq 0$$

$$\text{ME: } i = 2$$

$$a^{N+|y|} b^{N+1} \in L \text{ BY P.L.}$$

$$\text{BUT } N+|y| > N+1, \text{ where } |y| > 0$$

$$\text{SO } a^{N+|y|} b^{N+1} \notin L$$

✗  
SO  $L$  IS NOT REGULAR

- 4 9. Analyze the following language,  $L$ , proving it is **non-regular** by showing that there are an **infinite** number of equivalence classes formed by the relation  $R_L$  defined by:

$$x R_L y \text{ if and only if } [\forall z \in \{a,b\}^*, xz \in L \text{ exactly when } yz \in L].$$

where  $L = \{ a^i b^j \mid i < j \}$ ,

You don't have to present all equivalence classes, but you must demonstrate a pattern that gives rise to an infinite number of classes, along with evidence that these classes are distinct from each other.

CONSIDER EQUIV CLASSES  $[a^i]_{R_L}$

$$a^i b^{i+1} \in L$$

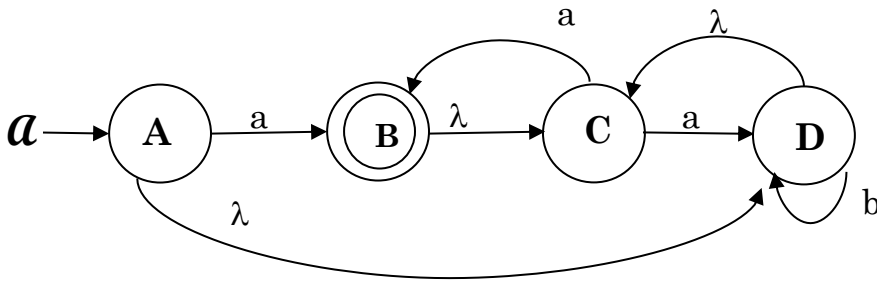
$$\text{BUT } a^i b^{j+1} \notin L \text{ FOR ALL } j > i$$

$$\text{SO } [a^i] \neq [a^j] \quad j > i$$

AND HENCE EACH SUCH CLASS DIFFERS  
AND THERE ARE AN INFINITE NUMBER OF  
SUCH CLASSES

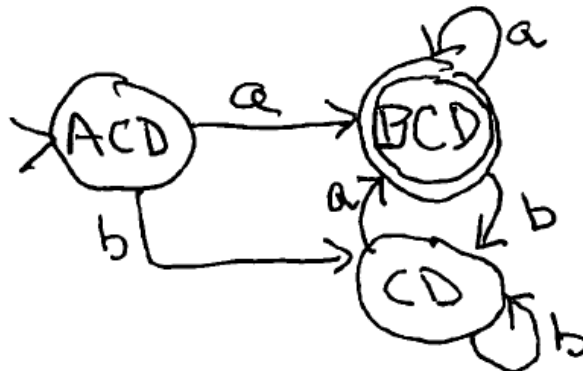
10. Let  $L$  be defined as the language accepted by the finite state automaton  $\mathcal{A}$ :

2 a.) Fill in the following table, showing the  $\lambda$ -closures for each of  $\mathcal{A}$ 's states.



State	A	B	C	D
$\lambda$ -closure	A, C, D	B, C	C	C, D

4 b.) Convert  $\mathcal{A}$  to an equivalent deterministic finite state automaton. Use states like AC to denote the subset of states  $\{A,C\}$ . Be careful --  $\lambda$ -closures are important.



3 11. Present a Mealy Model finite state machine that reads an input  $x \in \{0, 1\}^*$  and produces the binary number that represents the result of adding the two complement representation of decimal -2, that is adding binary  $1...1110$  to  $x$  (this assumes all numbers are in two's complement notation, including results). Assume that  $x$  is read starting with its least significant digit.

Examples:  $00010 \rightarrow 00000$ ;  $11001 \rightarrow 11010$ ;  $00001 \rightarrow 11111$



- 7 12. Given a DFA denoted by the transition table shown below, and assuming that **1** is the start state and **3** and **6** are final states, fill in the equivalent states matrix I have provided. Use this to create an equivalent, minimal state DFA.

	a	b	c
>1	4	6	5
2	1	6	5
<u>3</u>	2	3	3
4	1	3	5
5	1	3	6
<u>6</u>	1	3	6

2	1,4				
<u>3</u>	X	X			
4	3,6	3,6	X		
5	1,4 3,6 5,6 X	3,6 5,6 X	X	5,6X	
<u>6</u>	X	X	1,2	X	X
	1	2	<u>3</u>	4	5

Don't forget to construct and write down your new, equivalent automaton!! Be sure to clearly mark your start state and your final state(s). In your minimum state DFA, label merged states with the states that comprise the merge. Thus, if **1** and **3** are indistinguishable, label the merged state as **13**.

{1,2,4} {3,6} {5}

