ASSIGN #7 KEY

1a) SHOW $L = \{a^i b^j \mid j > i^3, i > 0\}$
IS NOT A CFL

ASSUME L IS A CFL

P.L.: $N > 0$

ME: $a^N b^{N^3+1} \in L$

P.L. $a^N b^{N^3+1} = uvwxy$, $|vwx| \leq N$, $|vx| > 0$
& $\forall i \geq 0$ $uv^iwx^iy \in L$

ME: • CHOOSE $i = 2$ IF vwx CONTAINS ANY a's
IF REPLICATION OF a's AND, PERHAPS, b's
OCCUR THEN WE WOULD ADD AT LEAST
ONE a AND AT MOST $N-1$ b's
THUS, $\#a\text{'s} \geq N+1$, $\#b\text{'s} \leq N^3+N$
BUT THERE MUST BE AT LEAST
$(N+1)^3+1$ b's FOR THIS STRING TO BE IN L
$(N+1)^3+1 = N^3+3N^2+3N+2 > N^3+N$
SO WE HAVE TOO FEW b's AND $uv^2wx^2y \notin L$

• CHOOSE $i = 0$ IF vwx CONTAINS NO a's
WE THEN HAVE N a's AND AT MOST
N^3 b's. AGAIN, THERE ARE TOO FEW
b's, SO $uv^0wx^0y = uwy \notin L$

CONCLUSION: L IS NOT A CFL

ASSIGN # 7 KEY

1 b) SHOW $L = \{a^i b^j \mid j < 3*i, i > 0\}$
IS A CFL

LET $G = (\{S, T, B\}, \{a, b\}, R, S)$

R: $S \rightarrow aSbbb \mid aTB$

$T \rightarrow aTB \mid \lambda$

$B \rightarrow b \mid bb \mid \lambda$

IN S, WE HAVE NO MORE THAN 3 b'S FOR EACH a IF RECURSE ON S. ONCE WE CHOOSE aTB, WE GUARANTEE FEWER THAN 3 b'S FOR AT LEAST ONE a.

THUS, $L(G) = L$

ASSIGN #7 KEY

2. $G = (\{S\}, \{a, b\}, R, S)$

$R: S \to aSbSbS \mid bSaSbS \mid bSbSaS \mid \lambda$

PROVE $L(G) = L = \{w \mid |w|_b = 2|w|_a\}$

THAT IS, w CONTAINS TWICE AS MANY b's AS a's

$L(G) \subseteq L$

CAN DO A DIRECT PROOF BY NOTING ALL RHS OF S HAVE TWICE AS MANY b's AS a's SO $w \in L(G) \Rightarrow w \in L$

NOW, SHOW

$L \subseteq L(G)$

DO BY LENGTH OF STRING IN L. AS ALL STRINGS IN L ARE $3k$, FOR SOME $k \geq 0$, WE HYPOTHESIZE THAT ALL STRINGS, w, OF LENGTH $3k$, $k \geq 0$, FROM L ARE IN $L(G)$

BASIS: $k = 0$. THEN $w = \lambda$ AND $S \to \lambda$ SO, $S \Rightarrow \lambda$ AND $\lambda \in L(G)$

IH: ASSUME FOR ALL k, $0 \leq k \leq m$ EVERY WORD OF LENGTH $3k$ IN L IS ALSO IN $L(G)$.

PROBLEM 2 CONTINUED

IS: SHOW THAT IF $|w| = 3(m+1)$ AND $w \in L$, i.e., HAS TWICE AS MANY b'S AS a'S THEN $w \in L(G)$.

THERE ARE THREE CASES
$w = a w'$ WHERE $w' = w_1 b w_2 b w_3$
AND EACH OF w_1, w_2 AND w_3 ARE IN L
THAT IS, THERE ARE TWO b'S MATCHED TO THE STARTING a, AND THESE b'S ARE SURROUNDED BY PARTS THAT ARE IN L AND EACH SUCH PART HAS LENGTH $\leq 3m$ BY IH AND THE FIRST RHS OF S,
$S \Rightarrow aSbSbS \overset{*}{\Rightarrow} a w_1 b w_2 b w_3$
AND SO $w \in L(G)$
THE REMAINING CASES START WITH b, WITH ONE GETTING THE MATCHING a BEFORE THE SECOND COUNTERBALANCING b, AND THE GETTING THE COUNTER-BALANCING b THEN THE MATCHING a.
CASES ARE
$S \Rightarrow bSaSbS \overset{*}{\Rightarrow} b w_1 a w_2 b w_3$
& $S \Rightarrow bSbSaS \overset{*}{\Rightarrow} b w_1 b w_2 a w_3$