Assignment # 5

1. For each of the following, prove it is not regular by using the Pumping Lemma or Myhill-Nerode. You must do at least one of these using the Pumping Lemma and at least one using Myhill-Nerode.

a. $\{ a^{2^k+1} \mid k \geq 0 \}$ (note: 2^k+1, so get $\{a^2, a^3, a^5, a^9, a^{17}, \ldots \}$)

b. $\{ a^i b^j c^k \mid i \geq 0, j \geq 0, k \geq 0,$ if $i=0$ then $j=2k \}$

c. $\{ xyz \mid x,y,z \in \{a, b\}^*$ and $y = xz \}$

2. Write a regular (right linear) grammar that generates the set of strings denoted by the regular expression $(((01 + 10)^+)(11))^* (00)^*$. You may use extended grammars where rules are of form $\mathbf{A} \rightarrow \alpha$ and $\mathbf{A} \rightarrow \alpha\, \mathbf{B}$, $\alpha \in \Sigma^*$ and **A**,**B** non-terminals

3. Write a Mealy finite state machine that produces the 2's complement result of subtracting 1101 from a binary input stream (assuming at least 3 bits of input)

Assignment # 5.1

1. For each of the following, prove it is not regular by using the Pumping Lemma or Myhill-Nerode. You must do at least one of these using the Pumping Lemma and at least one using Myhill-Nerode.

a. { $a^{2^\wedge k+1}$ | k≥0 } (note: 2^k+1, so get {a^2, a^3, a^5, a^9, a^{17}, … })

b. { $a^i b^j c^k$ | i≥0, j≥0, k≥0, if i=0 then j=2k }

c. { xyz | x,y,z ∈ {a, b}* and y = xz }

Assignment # 5.1 Answer

1a. $\{a^{2^k+1} \mid k \geq 0\}$ using P.L.

1. Assume that **L** is regular
2. Let **N** be the positive integer given by the Pumping Lemma
3. Let **s** be a string $s = a^{2^N+1} \in L$
4. Since $s \in L$ and $|s| \geq N$, **s** is split by PL into **xyz**, where $|xy| \leq N$ and $|y| > 0$ and for all $i \geq 0$, $xy^iz \in L$
5. We choose $i = 2$; by PL: $xy^2z = xyyz \in L$
6. Thus, $a^{2^N+1+|y|}$ would be in **L**. This means that there is number in **L** between a^{2^N+1} and $a^{2^N+1+N|}$, but next number after **2^N+1** is **2^(N+1)+1**. The distance is 2^N between these number and 2^N is greater than N for all values of N, meaning $a^{2^N+1+|y|}$ cannot be in **L.**
   This is a contradiction, therefore L is not regular ■

1b. $\{ a^ib^jc^k \mid i\geq0, j\geq0, k\geq0, \text{if } i=0 \text{ then } j=2k \}$ using P.L.

1. Assume that **L** is regular
2. Let **N** be the positive integer given by the Pumping Lemma
3. Let **s** be the string $s = b^{2N}c^N \in L$
4. Since $s \in L$ and $|s| \geq N$, **s** is split by PL into **xyz**, where $|xy| \leq N$ and $|y| > 0$ and for all $i \geq 0$, $xy^iz \in L$
5. We choose $i = 0$; by PL: $xy^0z = xz \in L$
6. Thus, $^{|}b^{2N-|y|}c^N$ would be in L, but it's not since $2N-|y| < 2N$
7. This is a contradiction, therefore **L** is not regular ■

Assignment # 5.1 Answer

1c. **{ xyz | x,y,z ∈ {a, b}* and y = xz }** using P.L.

1. Assume that **L** is regular
2. Let **N** be the positive integer given by the Pumping Lemma
3. Let **s** be the string $\mathbf{s = a^N b a^N b \in L}$
4. Since $\mathbf{s \in L}$ and $\mathbf{|s| \geq N}$, **s** is split by PL into **xyz**, where $\mathbf{|xy| \leq N}$ and $\mathbf{|y| > 0}$ and for all $\mathbf{i \geq 0}$, $\mathbf{xy^i z \in L}$
5. We choose **i = 0**; by PL: $\mathbf{xy^0 z = xz \in L}$
6. Thus, $\mathbf{a^{N-|y|} b a^N b}$ would be in **L**.
   One **b** has to be part of **x** and the other of **y**, or of **y** and **z.** If one **b** in **x** then, since $\mathbf{N-|y| \neq N}$, this is not of the proper form. If the **b**'s are in **y** and **z**, then we ecounter the same issue.
7. This is a contradiction, therefore L is not regular ■

Assignment # 5.1 Answer

1a. $\{a^{2^k+1} \mid k \geq 0\}$ using M.N.

We consider the collection of right invariant equivalence classes $[a^{2^i+1}]$, $i \geq 0$.

It's clear that $a^{2^i+1} a^{2^i} = a^{2^{(i+1)}+1}$ is in the language, but $a^{2^j+1} a^{2^i} = a^{2^j+2^i+1}$ is not as 2^j+2^i is not a power of two when $i \neq j$. To see this, assume wlog that $j>i$, then the next power of two after 2^j is $2^{(j+1)} = 2^j + 2^j > 2^j + 2^i$.

This shows that there is a separate equivalence class $[a^{Fib(j)}]$ induced by R_L, for each $j > 2$.

Thus, the index of R_L is infinite and Myhill-Nerode states that L cannot be Regular. ■

1b. $\{ a^i b^j c^k \mid i \geq 0, j \geq 0, k \geq 0, \text{ if } i=0 \text{ then } j=2k \}$ using M.N.

We consider the collection of right invariant equivalence classes $[b^{2i}]$, $i \geq 0$.

It's clear that $b^{2i}c^i$ is in the language, but $b^{2j}c^i$ is not when $j \neq i$

This shows that there is a separate equivalence class $[b^{2i}]$ induced by R_L, for each $i \geq 0$.

Thus, the index of R_L is infinite and Myhill-Nerode states that L cannot be Regular. ■

1c. $\{ xyz \mid x,y,z \in \{a, b\}^* \text{ and } y = xz \}$ using M.N.

We consider the collection of right invariant equivalence classes $[a^j b]$, $i \geq 0$.

It's clear that $a^j b a^j b$ is in the language, but $a^j b a^i b$ is not when $j \neq i$

This shows that there is a separate equivalence class $[a^j b]$ induced by R_L, for each $i \geq 0$.

Thus, the index of R_L is infinite and Myhill-Nerode states that L cannot be Regular. ■

Assignment # 5.2

2. Write a regular (right linear) grammar that generates the set of strings denoted by the regular expression $(((01 + 10)^+)(11))^* (00)^*$. You may use extended grammars where rules are of form **A** $\rightarrow$ α and **A** $\rightarrow$ α **B**, $\alpha \in \Sigma^*$ and **A**,**B** non-terminals

G = ({S,T,U,V}, {0,1}, S, P)

P:

S $\rightarrow$ **T | V**

T $\rightarrow$ **01T | 10T | 01U | 10U**

U $\rightarrow$ **11S**

V $\rightarrow$ **00V |** λ

Assignment # 5.3

Write a Mealy finite state machine that produces the 2's complement result of subtracting 1101 from a binary input stream (assuming at least 3 bits of input)

Answer