# Assignment # 5.1 (M-N)

a. $L = \{ x\#y \mid x, y \in \{0,1\}^+$ and $y$ is the ones complement of $x \}$

Let $R_L$ be the right invariant equivalence class defined by M-N for L. Consider the pair of equivalent classes $[010^i\#]$ and $[010^j\#]$, $i \neq j$, $i,j>0$. $010^i\#101^i \in L$ but $010^j\#101^i \notin L$. Thus, for each distinct pair, i,j, $i \neq j$, $[010^i] \neq [010^j]$ and hence $R_L$ has infinite index.

L is not Regular by Myhill-Nerode Theorem.

# Assignment # 5.1 (PL)

a. $L = \{ x\#y \mid x, y \in \{0,1\}^+$ and $y$ is the ones complement of $x \}$

PL: Gives me N>0 associated with L
Me: Choose $w=10^N\#01^N$ which is in L
PL: States $w=xyz$, $|xy|\leq N$, $|y|>0$, $xy^iz \in L$ for all $i\geq 0$
Me: Choose i = 0.
Case 1: If the initial 1 is erased, we have $0^{N-|y|-1}\#01^N$ which is not in L.
Case 2: If only 0's are affected, we have $10^{N-|y|}\#01^N$, which is not in L when $|y|>0$ (would need to end with $\#01^{N-|y|}$).

Thus, L is not Regular by Pumping Lemma for Regular Languages.

# Assignment # 5.1b (M-N)

b. $L = \{ a^i b^j c^k \mid i > j*k \}$

Let $R_L$ be the right invariant equivalence relation defined for L by M-H.
Consider $[a^{i+1}b^i]$ and $[a^{i+1}b^j]$, $j > i$.
$a^{i+1}b^i c \in L$ $(i+1 > i*1)$
but $a^{i+1}b^j c \notin L$ $(i+1 \le j*1$ as $j>i$ and so $j*1$ is at least $i+1)$

Thus, for any two distinct i,j, $i \neq j$, $[a^{i+1}b^i] \neq [a^{i+1}b^j]$

# Assignment # 5.1b (PL)

b. $L = \{ a^i b^j c^k \mid i > j*k \}$.

Me: L is regular
PL: Gives me N>0 associated with L
Me: Choose $w=a^{N+1}b^Nc$ which is in L
PL: States $w=xyz$, $|xy|\le N$, $|y|>0$, $xy^iz \in L$ for all $i\ge 0$
Me: Choose i = 0. This says that $xz = a^{N-|y|+1}b^Nc \in L$. But, since $|y|>0$, then $N-|y|+1 \le N$, but the number of b's (N) times the number of c's (1) is N and so the number of a's is not sufficient.

Thus, L is not Regular by Pumping Lemma for Regular.

# Assignment # 5.1c (M-N)

c. L = { x w x | x, w $\in$ {a,b}$^+$ Here |x|>0 and |w|>0 }

**I attack this with M-N. Let $R_L$ be the right invariant equivalence relation defined for L by M-H.**
**Consider [$a^ib$] and [$a^jb$] i ≠ j.**
**$a^iba^i \in$ L but $a^jba^i \notin$ L, when j ≠ I (b must be the w part and so the x part is all a's before the b and must must the number of a's after the b..**
**Thus, for any two distinct i,j, i≠j, [$a^ib$] ≠ [$a^jb$].**

**Thus, L is not Regular by Myhill-Nerode Theorem.**

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# Assignment # 5.1c (PL)

**c. $L = \{ x\, w\, x \mid x, w \in \{a,b\}^+ \}$**

**PL: Gives me N>0 associated with L**
**Me: Choose $v=a^N b a^N$ which is in L**
**PL: States $v=xyz$, $|xy|\leq N$, $|y|>0$, $xy^iz \in L$ for all $i\geq 0$**
**Me: Choose i = 0. $a^{N-|y|}ba^N \notin L$ since a's before b do match those after b.**

**Thus, L is not Regular by Pumping Lemma for Regular Languages.**

# Assignment # 5.2

2. Write a regular (right linear) grammar that generates **L = { w | w ∈ {0,1}$^+$** and **w** interpreted as a binary number is divisible by 2 or 3 or both}.

**<0> → 0 <0> | 1 <1> | λ**

**<1> → 0 <2> | 1 <3>**

**<2> → 0 <4> | 1 <5> | λ**

**<3> → 0 <0> | 1 <1> | λ**

**<4> → 0 <2> | 1 <3> | λ**

**<5> → 0 <4> | 1 <5>**

**Can reduce to five rules as we showed a 5 state DFA recognizes this language. The trick is to combine rules <0> and <3>, as they do the exact same thing.**

# Assignment # 5.3

2. Present a Mealy Model finite state machine that reads an input **x** $\in$ **{0, 1}**$^+$ and produces the binary number that represents the result of adding binary **1001** to **x** (assumes all numbers are positive, including results). Assume that **x** is read starting with its least significant digit.
Examples: **00010** $\rightarrow$ **01011; 00101** $\rightarrow$ **01110; 00111** $\rightarrow$ **10000; 00110** $\rightarrow$ **01111**